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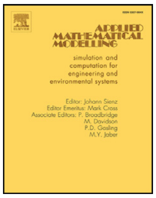

An analytical solution to the thermal problems with varying boundary conditions around a moving source

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ABSTRACT

This article devises an analytical solution to the practical industrial applications, in which the thermal boundary conditions are varying in both time and space around a cooling or heating source moving along a finite domain. Among the various applications of such a solution, the jet impingement cooling and underwater welding and cutting would be of great importance. Afterwards, a numerical example regarding the temperature field caused by an underwater solid state welding process is provided by using the developed analytical solution. Finally, in order to evaluate the accuracy of the presented analytical model, the analytically obtained results are compared with the results attained by FEM-Comsol commercial software.

1. Introduction

Analytical solutions to the temperature field over a finite plate with boundary conditions (BC) varying in time and space allow one to obtain the temperature distribution around a cooling or heating source moving along a plate with BC correlating with the location of the source. Thus, finding such an analytical model would be a viable and resilient tool for wide industrial applications including jet impingement cooling and under water welding as well as cutting processes.

For instance, in a jet impingement cooling, a high-velocity fluid is vertically directed onto the surface to be cooled. As the fluid hits the impingement surface, the fluid is diverted in all directions, parallel to the surface. The cooling rate caused by this method is high, but declines as the distance from the jet is increased. Thus, to ensure effective cooling on a larger zone, either abundant impingement jets are utilized or impingement jets are moved along the surface to be cooled, which represent problems with non-uniform BC varying in time.

Application of varying BC is not limited to the jet impingement cooling and can be extended to some other industrial applications, one of which is underwater welding and cutting process. Harbor clearance, underwater pipelines, and conveyer equipment repair need considerable underwater cutting and welding, which itself is classified as solid state and conventional welding. Generally, during the underwater welding or cutting, since the workpiece temperature increases beyond the water saturation point, different boiling regimes are expected through the workpiece, representing non-uniform heat convection around the welding or cutting beam. Whilst as the welding or cutting beam moves, the convention BC varies in both time and space.

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Thus, the provided brief explanation regarding the various applications of a solution to the temperature field over a plate with BC varying in time and space and subjected to a moving cooling or heating source, accentuates the importance of such an analytical solution.

Due to the widespread applications of the heat transfer equation in various industrial processes such as welding, machining and rolling, various methods have been devised to analytically solve it under different geometrical and boundary conditions. The most highlighted innovations in these studies have been focused on heat source definitions including volumetric, tilted and Gaussian power distribution heat sources which move along a linear or an elliptical trajectory regarding their particular industrial applications [1–4]. In some other investigations, it has been also improved the convergence of the problem using some common solution procedures. Despite the wide variety of proposed analytical solutions for a moving heat source, none of them is able to consider an arbitrary varying function for boundary conditions [5].

Among the employed analytical methods for moving heat source problems, Integral Transformation provides the minutest restriction and was employed by Olcer [6] and Ozisik [7] for the first time. Furthermore, one of the most comprehensive and fundamental studies carried out on this field can be attributed to Mikhailov [8], who extended the solution devised by Olcer to a more general form, applicable for the prediction of heat transfer in laminar and turbulent flows of Newtonian and non-Newtonian fluids. One year later, Mikhailov [9] introduced a new finite integral transform for the solution of diffusion equations with coupled BC. The coefficients of the problem were assumed so that the solution of Sturm–Liouville problem became only a function of space variable. Subsequently, Ozisik and Murray [10] complemented Mikhailov's solution so that the convection coefficient became a function of both time and space variables. Then, Mikhailov [11] tackled the complexity of the time functionality not only for the heat transfer coefficient but also for all of the coefficients involved in the problem. In both studies the integral transform method was implemented and the final solutions were presented in terms of related eigen-functions. Expectedly, in both studies, eigen-functions and values were time-dependent due to the considered time-dependent convection coefficient and consequently constrained the application of the proposed solution to the solution of time-dependent Sturm–Liouville problem in its general form. However, the solution to such a boundary value problem is available only for some limited special cases and would be too complicated and might be even impossible for some other cases. Thus, to avoid the complications of time-dependent Sturm–Liouville problem, Cotta [12] considered the time dependency for one of the coefficients of the main heat equation or BC and decomposed the coefficient into two parts: the first part, which is only a function of time and the second part, which is a function of both time and space. He also described some special conditions, which facilitate the use of simplifying assumptions. His proposed solution appears to cover a wide range of applications with high accuracy.

Hence, according to the reviewed literature, due to the great applications of analytical mathematical solutions to the temperature field during industrial processes a considerable effort has been devoted to translate the thermal condition of engineering processes from the real world into mathematical language. This effort has been continuing up to the last years, some of which were to employ the theoretical mathematical solutions to model practical engineering applications such as welding processes, which has been earlier noted in the current article. In this regard, in 2008, Kukla [2] presented an analytical model to a temperature field caused by a rectangular heating source moving along a finite plate by using Green's Function. Afterward, in 2011, Kim [13] used the Fourier series to apply more governing parameters of the temperature distribution around a rectangular moving heating source. In the same year, Fernandes, et al. [14] employed Green's function to decrease computational time in inverse heat conduction problems and showed that the analytical solution based on the Green's function to predict the temperature distribution in heat conduction problems not only enhances the precision, but also decreases considerably the computational time.

Similarly, in 2013, the authors [15] devised an analytical solution to the temperature field around a heat source moving along a finite plate with non-homogeneous boundary conditions and non-uniform initial temperature. Also, in 2014, they [16] employed the finite integral transform introduced by Mikhailov to develop a transient analytical solution to the heat conduction problem in dissimilar plates butted together and subjected to a circular moving heat source as a welding tool or beam. The results obtained by their model exhibited great agreement with the experimental data, which accentuated the accuracy of the devised analytical solution. Finally, in 2015, they [17] extended the previous analytical solution [15] to cover the application of non-uniform heat source. Thus, the recent studies have focused on using the pure mathematical methods in real practical engineering applications and translating the official language of complex equations to the casual language of results, contours and figures.

Similarly, having the work done by Cotta in view, the current study provides a transient analytical solution to the heat conduction problem characterized by a moving circular convection boundary condition and also a moving circular heat source. The solution would be a viable response to the needs, addressed earlier in this article, regarding finding thermal analytical models for engineering processes such as jet impingement cooling and underwater welding.

In other words, in order to complement the previous studies, the solution to be found in this article addresses a varying convection BC, which is circularly applied around a cooling or heating source and move at the same velocity as the source moves. As illustrated in Fig. 1, each circle, located in the vicinity of the source, represents a different convection BC, which decrease by increasing the distance from the impinging jet (a) or similarly are caused by different boiling regimes around the underwater welding and cutting beam (b).

Furthermore, as a numerical example, an underwater solid state welding is simulated by the developed analytical solution and subsequently the analytically computed results are compared with those obtained by a numerical solution carried out by commercial software to evaluate the accuracy of the developed analytical model.