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The role of magnetic Reynolds number in MHD forced convection heat transfer

S. Vimala[a], S. Damodaran[b], R. Sivakumar[c], T.V.S. Sekhar[d,*]

[a] Department of Mathematics, Pondicherry Engineering College, Puducherry 605014, India
[b] Department of Mathematics, Bharathidasan Government College for Women, Puducherry 605003, India
[c] Department of Physics, Pondicherry University, Puducherry 605014, India
[d] School of Basic Sciences, Indian Institute of Technology Bhubaneswar, Bhubaneswar 751007, India

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ABSTRACT

Higher Order Compact (HOC) Scheme combined with multigrid method is developed to investigate the role of magnetic Reynolds number in MHD heat transfer from a circular cylinder which is under the influence of an external magnetic field. The governing equations are coupled nonlinear Navier–Stokes and nonlinear Maxwell's equations which are expressed in stream function, vorticity and magnetic stream function (ψ-ω-A) formulation and decoupled energy equation. Apart from the usual kinetic Reynolds number *Re*, the parameters that governs the flow are magnetic Reynolds number R_m, Alfvén number β and Prandtl number *Pr*. The velocities of the flow are calculated with second order accurate based finite difference scheme and heat transfer equation is solved with fourth order accurate HOC scheme. The influence of the magnetic field and magnetic Reynolds number on velocity gradients is presented. It is found that the mean Nusselt number decreases until $N \leq 1$ and then increases with further increase in the interaction parameter. It is also found that fluids having higher electrical conductivity can be effectively controlled with relatively low magnetic fields. The decrease in mean Nusselt number with β and R_m is in agreement with experimental findings.

1. Introduction

It is well known that the viscous flows past a bluff body will create a wake and induce a vortex street behind the body. This feature will be accompanied by changes in drag and lift forces. These are undesirable flow features as they are harmful to structural vibrations and acoustic noise. These phenomena may be suppressed through flow control methods such as rotary oscillation [1], sound wave disturbance [2], the thermal effect [3], setting a second cylinder in downstream in cruciform arrangement [4], suction and blowing [5], etc. The application of Lorentz force can control the wake behind the bluff body and it has attracted the attention of researchers recently due to sound engineering applications. The Lorentz force application on the flow can be broadly divided in to two cases. Firstly the widely used quasi-static approximation (or low R_m approximation where R_m is magnetic Reynolds number) in which the induced magnetic field is neglected for very low magnetic Reynolds numbers ie $R_m < <1$ (QS model). Here the magnetic field becomes constant and hence many

* Corresponding author. Tel.: +91 6742576077.
E-mail addresses: vimalaks@pec.edu (S. Vimala), sreedamo@gmail.com (S. Damodaran), rsk.phy@pondiuni.edu.in (R. Sivakumar), sekhartvs@iitbbs.ac.in, sekhartvs@yahoo.co.in (T.V.S. Sekhar).

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non-linearities that arise due to Ohms law can be eliminated. Some notable theoretical, computational and experimental works are [6–15]. Secondly, for moderate and high magnetic Reynolds numbers, the induced field cannot be neglected and hence the need to solve full Navier-Stokes equations coupled with Maxwell's equations (full MHD model). Unfortunately there are few papers in this direction out of which some of them are [16–20].

MHD flows have tremendous engineering importance when treated with heat transfer problems. They are not trivial and hence require vast investigation, especially since the Maxwells electromagnetic equations when coupled with traditional fluid dynamics, generate additional body force, viz., the Lorentz forces. The Lorentz force opposes fluid motion across magnetic field lines thereby slowing the flow near the surface of the body and reducing heat transfer and skin friction. Gardner and Lykoudis [21] experimentally studied the transverse magnetic field effects on the heat transfer and concluded that it inhibits the convective mechanism of heat transfer resulting in the reduction of Nusselt number. Blum [22] conducted heat transfer experiment using an electrolyte flowing through a rectangular channel over a wide range of Reynolds numbers including the transition region from laminar to turbulent and presented the degradation of heat transfer as a function of the interaction parameter. A reduction of heat transfer rate [23] and an increase in drag [24,25] were also experimentally found in magnetogasdynamics. The transverse magnetic field effects on forced convection heat transfer of liquid lithium flowing in an annular channel were studied by Uda et al. [26] experimentally and found the decrease in Nusselt number with interaction parameter. Genin et al. [27] experimentally investigated transverse magnetic field effects on heat transfer for liquid metal coolant flowing through a horizontal heated tube for a wide range of Reynolds and Hartman numbers. In their experimental study Yokomine et. al [28] investigated turbulent heat transfer effects of high Prandtl number fluid flow under a strong magnetic field and reported that there is a degradation of heat transfer. The forced convective heat transfer for the MHD flow under QS model has been studied recently [29,30].

Higher Order Compact Schemes (HOCS) are becoming more popular in recent days due to their higher order accuracy in results in addition to unconditional stability of the numerical scheme. These schemes capture the flow phenomena very accurately in coarser grids itself when compared to upwind schemes and also some second order accurate finite difference schemes [31–33]. In this paper, the effect of magnetic Reynolds number on the forced convection heat transfer from a circular cylinder in the presence of applied magnetic field is investigated by using HOCS.

2. Formulation of the problem

The equations governing the motion (left to right) of an electrically conducting fluid past a perfectly conducting circular cylinder with a uniformly applied magnetic field in the direction of the flow at large distances are, in non-dimensional form,

$$(\boldsymbol{q}\cdot\nabla)\,\boldsymbol{q} = -\nabla p + \frac{2}{Re}\nabla^2\boldsymbol{q} + \frac{2N}{R_m}\boldsymbol{j}\times\boldsymbol{H} \tag{1}$$

$$\boldsymbol{j} = \nabla\times\boldsymbol{H} = \frac{R_m}{2}[\boldsymbol{E}+\boldsymbol{q}\times\boldsymbol{H}] \tag{2}$$

$$\nabla\cdot\boldsymbol{q} = 0 \tag{3}$$

$$\nabla\cdot\boldsymbol{H} = 0 \tag{4}$$

$$\nabla\times\boldsymbol{E} = 0 \tag{5}$$

where p is the pressure, $\boldsymbol{q}$ the fluid velocity, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{E}$ the electric field, $\boldsymbol{j}$ the current density. The kinematic Reynolds number $Re = 2aU/\nu$ (a is the radius of the circular cylinder), N is the interaction parameter defined as $N = \sigma H_\infty^2 a/\rho U_\infty$ and magnetic Reynolds number $R_m = 2aU\mu\sigma$. Here U is the uniform flow and H is the magnitude of the applied magnetic field. The kinematic viscosity, density, magnetic permeability and electrical conductivity of the fluid are ν, ρ, μ and σ respectively. By taking curl of the momentum Eq. (1) and using the Eq. (2) we get

$$\nabla^2\boldsymbol{\omega} = \frac{Re}{2}\nabla\times(\boldsymbol{\omega}\times\boldsymbol{q}) - \frac{ReN}{2}\nabla\times\{(\boldsymbol{q}\times\boldsymbol{H})\times\boldsymbol{H}\} \tag{6}$$

$$\boldsymbol{\omega} = \nabla\times\boldsymbol{q} \tag{7}$$

is the vorticity. Since the flow is two dimensional, $\boldsymbol{E} = 0$. Cylindrical polar co-ordinates (r, θ, z) are used as they are most suitable in dealing with cylindrical boundaries.

The co-ordinate system is set up such that the velocity and magnetic field are parallel at large distances and the flow is symmetric about $\theta = 0^o$ and $\theta = 180^o$. Here, $\boldsymbol{q} = (q_r, q_\theta, 0)$, $\boldsymbol{H} = (h_r, h_\theta, 0)$, $\boldsymbol{j} = (0, 0, j)$. In order to satisfy Eqs. (3) and (4), the dimensionless stream function $\psi(r, \theta)$ and magnetic stream function $A(r, \theta)$ are introduced such that

$$q_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad q_\theta = -\frac{\partial\psi}{\partial r}, \quad h_r = \frac{1}{r}\frac{\partial A}{\partial\theta}, \quad h_\theta = -\frac{\partial A}{\partial r}.$$

We use the transformation $r = e^{\pi\xi}$ and $\theta = \pi\eta$, so that the governing Full MHD equations in the stream function-vorticity-magnetic stream function (ψ-ω-A) formulation [19] are as follows: