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Multi-class time reliability-based congestion pricing model based on a degradable transportation network

Bing-quan Liu [a,b,*], Chong-chao Huang [a]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
[b] College of Mathematics and Information Science, Weinan Normal University, Weinan 714099, China

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ABSTRACT

In this study, we present a multi-class time reliability-based user equilibrium model with elastic demand, which is based on a degradable transportation network. Due to the uncertainties in the path travel time induced by random capacity degradation, the model captures the path choice behaviors of travelers in the form of the robust effective path travel time and there is no need for a known travel time probability distribution. By including the heterogeneous degrees of risk aversion among travelers, the equilibrium model can be transformed into an equivalent variational inequality (VI) problem. We also propose a heuristic solution algorithm to solve the VI problem. By taking the VI model as a constraint, a multi-class time reliability-based congestion pricing model is formulated as a mathematical programming with equilibrium constraints problem, which can be solved using the sensitivity analysis-based conjugate sub-gradient projection method. Two numerical examples are provided to illustrate the applications of the proposed models and to demonstrate the efficiency of the solution algorithms.

1. Introduction

1.1. Motivation

It is widely recognized a transportation network is an uncertain system due to diverse sources of variability. These uncertainties can be categorized into two main classes: supply stochasticity and demand fluctuations. The supply stochasticity, i.e., random link capacity degradation, is caused by different disturbances on the road network, such as traffic incidents, traffic management and control, illegal parking, weather conditions, or even floods and earthquakes. Demand fluctuations can also be observed in the daily variations in the demand flow. These supply and demand uncertainties can seriously affect the driving conditions and link capacity, thereby directly leading to unreliable travel time and traffic conditions. Naturally, these uncertainties cause undesirable delays for travelers. Therefore, travelers cannot know exactly when they will arrive at their destination. If travelers make their path choice decisions based only on the mean travel time, they may arrive late and incur a penalty. Several empirical surveys have shown that travelers are interested in reducing their travel time and enhancing the reliability of the travel time during their path choice decision processes [1,2].

In most modern metropolises, especially in the central business district, travelers usually experience serious traffic congestion, which is viewed as a major bottleneck for transportation sustainability. It is well known that congestion pricing is an efficient

* Corresponding author at: School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China. Tel.: +86 155 2751 6979.
E-mail address: lbingquan@eyou.com (B.-q. Liu).

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method for alleviating this problem. The link (path) flow and traffic demand can be affected by charging road users a suitable toll, thereby allowing the more efficient utilization of existing transportation systems, and even optimizing the system flow pattern. Undoubtedly, the uncertainties in transportation system can affect the traffic flow distribution, but they also influence the decisions of policymakers. After a congestion pricing scheme has been adopted, then according to the hypothesis of a deterministic transportation network, the network may become more congested and even worsen if the link flow and traffic demand are estimated incorrectly. Thus, it is essential to design a congestion pricing scheme for an uncertain transportation system, as described in the present study.

1.2. Literature review

In the last two decades, for uncertain transportation systems, studies of travel time reliability and the path choice behaviors of travelers have attracted increasing attention from researchers in various fields, e.g., engineers, mathematicians, and planners. Many models have been formulated to explicitly incorporate supply stochasticity or demand fluctuation into these analyses. On the supply side, Lo and Tung [3] developed a probabilistic user equilibrium model to capture the path choice behaviors of travelers. In their model, they described the link capacity as a uniformly distributed random variable, which is subject to daily random degradations. Lo et al. [4] also developed a multi-class mixed-equilibrium model based on the concept of the travel time budget. This model captures the path choice behaviors of travelers in form of the path travel time budget (PTTB). Shi and Luo [5] formulated a multi-class variational inequality (VI) model to describe the different path choice behaviors of travelers in a degradable network, where a heuristic solution algorithm was developed to solve the VI model. Siu and Lo [6] proposed a mixed user equilibrium model for a doubly uncertain network with random supply and demand, where they transformed the model into a nonlinear complementarity problem. Lam et al. [7] explicitly considered the impact of adverse weather conditions on the road performance and proposed a stochastic user equilibrium (SUE) model based on the effective path travel time.

On the demand side, Clark and Watling [8] were probably the first to model the problem of travel time reliability using the analysis method. Shao et al. [9] presented a reliability-based SUE model with heterogeneous user classes. In their model, each class of travelers hedges against travel time uncertainties by allowing a path travel time safety margin to ensure punctual arrival. Sun and Gao [10] employed the worst-case value-at-risk and worst-case conditional value-at-risk to define the robust percentile path travel time (RPPTT) and robust mean-excess path travel time (RMEPTT), thereby proposing a robust percentile SUE model that does not require the path travel time distribution. In their study, the RPPTT and RMEPTT performed equally well under general convex distributions. Shao et al. [11] proposed a travel time estimator to estimate the variable path travel time under demand uncertainty, and a bi-level programming problem was formulated to explicitly consider the reliability-based path choice behaviors of travelers. Recently, Fu et al. [12] also investigated the route choice behaviors of travelers using a reliability-based traffic model of a multi-modal transportation network under demand uncertainty.

To study aspects of the travel time stochasticity, some interesting time reliability-based models have been proposed to capture the path choice behaviors of travelers. For example, Chen and Zhou [13] introduced an α-reliable mean-excess stochastic travel time traffic equilibrium model, which differs from the PTTB model, where they explicitly considered the reliability and unreliability aspects of travel time stochasticity in the route choice behavior of travelers. Subsequently, they modeled the travel time stochastic perception error in the mean-excess traffic equilibrium model and they extended this model to heterogeneous risk aversion and elastic demand conditions [14,15]. Recently, Wang et al. [16] integrated the path travel time, travel time reliability, and total time budget into the user equilibrium problem, and presented a bi-objective user equilibrium model to explore the performance of the three path choice criteria. Tan et al. [17] investigated the Pareto efficiency of various time reliability-based traffic assignment models. Analyzing the Pareto efficiency can allow traffic planners to select an appropriate time reliability-based model for practical applications.

All of the studies mentioned above demonstrate that travelers cannot ensure their path travel time due to uncertainties in the transportation system. Hence, it is unreasonable to design a congestion pricing scheme based on a hypothetical deterministic transportation network because uncertainties can affect the traffic flow distribution and the decisions of planners. Gardner et al. [18] analyzed the robust congestion pricing scheme based on the marginal costs and evaluated three possible methods for determining the pricing scheme based on the demand in an uncertain network. They also presented two more practical methods, i.e., the single point approximation method and multiple point inflation/deflation approximation method, to solve the robust congestion pricing model [19]. Sumalee and Xu [20] described the first best marginal cost pricing scheme for a transportation network with stochastic travel demand, where they derived a closed-form formulation from the VI conditions for the optimum system and user equilibrium. Sun et al. [21] recently generalized the reliability-based network design problem with random traffic demand to advanced traveler information systems, where they formulated a bi-level programming model to maximize the reliability of the total travel time based on a general transportation network. However, relatively few studies have addressed the congestion pricing problem in an uncertain transportation network, especially for a capacity degradation network. Thus, in the present study, we propose a congestion pricing model for a degradable network.

1.3. Contribution statement

In this study, we propose a multi-class time reliability-based user equilibrium with elastic demand (MCRUE-ED) traffic assignment model to consider the daily capacity degradation. Travelers with heterogeneous degrees of risk aversion are considered in the transportation network. The robust effective path travel time (REPTT) is employed to capture the path choice behaviors of