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A boundary element method formulation for modal analysis of doubly curved thick shallow shells

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ABSTRACT

The study of vibrations of shells is an important aspect in the design of thin-walled structures. In general, analytical solutions for the natural frequencies of shells are not possible, and computational techniques are required. In this paper, modal analysis of shallow shells using a new boundary element method formulation is presented. The proposed formulation is based on a direct time-domain integration using the elastostatic fundamental solutions for both in-plane elasticity and shear-deformable plates. We modeled shallow shells by coupling the boundary element formulation of a shear-deformable plate and the two-dimensional plane stress elasticity. Effects of shear deformation and rotatory inertia were included in the formulation. Domain integrals related to inertial terms were treated by the dual reciprocity boundary element method. Numerical examples are presented to demonstrate the efficiency and accuracy of the proposed formulation.

1. Introduction

Since the 1960s, modal analysis of plates and shell structures has mainly been studied experimentally. However, geometric complexity and discontinuities of the structural systems under study make experimental modal analysis cumbersome [8,9,18]. Numerical models are the only way to obtain approximate solutions in modal analysis, thus playing a key role as a tool complementary to experimental modal analysis [12,13].

Traditional computational methods based on domain discretization such as the finite element method (FEM) and the finite difference method have been used extensively in vibrational analysis [10,11]. These methods can be used to analyze problems of vibration at low and medium frequencies. However, the FEM and the finite difference method become impracticable and expensive in terms of computer power and simulation cost at high frequencies because as the frequency increases, the wavelength becomes shorter and more elements are needed to create the model [2]. In the high-frequency range, advanced methods are already used for vibration analysis, such as statistical energy analysis [16], wave intensity analysis [15], the smooth energy method [4], and the power injection method [17].

Recently, mesh reduction methods, such as the boundary element method (BEM), have emerged as accurate and efficient numerical methods for plate and shell static analysis, [3,6,24,25,38,40]. The BEM has also been applied successfully to the dynamic analysis of plates [21]. BEM dynamic analysis of shear-deformable plates using elastodynamic fundamental solutions or Laplace or Fourier transformations of these fundamental solutions was used in [7,35–37]. Zhang and Atluri [41] and Providakis and Beskos [22] presented a BEM dynamic analysis of thin shallow shells based on a direct time-integration formulation. In [31,32]

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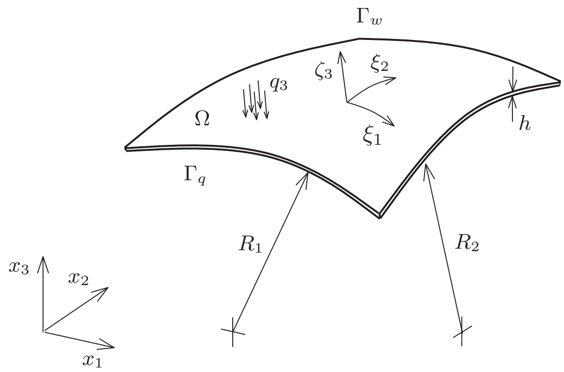

Fig. 1. Shallow shell geometry.

a time-domain direct formulations based on elastostatic fundamental solutions for dynamic analysis of shear-deformable plates is presented. Schweizerhof and Wang [33] presented a boundary element formulation, based on the cell integration method, for free vibration of a thick orthotropic laminated shallow shell.

Compared with elastodynamic fundamental solutions, the use of elastostatic fundamental solutions in BEM formulations has the advantage of mathematical simplicity, thus allowing the application of well-established methods to treat domain integrals [21]. There are a large number of studies in the literature in which elastodynamic problems have been analyzed successfully by the use of elastostatic fundamental solutions [1,23,28,30]. In these studies, domain integrals where treated by the cell integration method, the radial integration method, or the dual reciprocity method (DRM). The DRM is a powerful method for transforming domain integrals into boundary integrals, having advantages over the cell integration method and the radial integration method because they do not require domain discretization [20]. The DRM has been applied successfully for static and dynamic analysis of plates and shells, demonstrating the potential of the BEM for vibration analysis of continuous structures [5,14,19,39]. However, to the best of our knowledge, this method has not been used for modal analysis of moderately thick shallow shells. Modal BEM analysis of shells is an attractive research field increasing knowledge in computational structural dynamics.

This paper presents modal analysis of shear-deformable doubly curved shallow shells using a boundary element formulation. The proposed formulation is based on direct time integration and elastostatic fundamental solutions. Effects of shear deformation and rotatory inertia are included in the formulation. We modeled shells by coupling the boundary element formulation for shear deformable plates based on the Reissner plate theory and two-dimensional plane stress elasticity, as presented in [6,39]. The dual reciprocity BEM (DRBEM) for the treatment of domain integrals involving inertial mass was used. Numerical examples are presented and the results are compared with those obtained with use of both analytical and finite element solutions.

2. Shallow shell dynamic equations

Consider a shallow shell of uniform thickness h and mass density ρ, occupying the area Ω, in the x_1x_2 plane, bounded by the contour $\Gamma = \Gamma_w \bigcup \Gamma_q$ with $\Gamma = \Gamma_w \bigcap \Gamma_q \equiv 0$, as presented in Fig. 1. We modeled the dynamic bending response for the shallow shell by coupling the classical Reissner plate theory and the two-dimensional plane stress elasticity as presented in [38].

The equations of motion for an infinitesimal plate element without consideration of distributed loads are given by [26]

$$Dw_{1,\alpha\alpha} + \frac{D}{2}(1+\nu)(-w_{1,22} + w_{2,21}) - C(w_1 + w_{3,1}) = I_2\ddot{w}_1 + I_1\ddot{u}_1, \tag{1}$$

$$\frac{D}{2}(1+\nu)(w_{1,12} - w_{2,11}) + Dw_{2,\alpha\alpha} - C(w_2 + w_{3,2}) = I_2\ddot{w}_2 + I_1\ddot{u}_2, \tag{2}$$

$$C(w_{3,\alpha\alpha} + w_{1,1} + w_{2,2}) + q_3^* = I_1\ddot{w}_3, \tag{3}$$

$$Bu_{1,\alpha\alpha} + \frac{B}{2}(1+\nu)(-u_{1,22} + u_{2,21}) + B(\kappa_{11} + \nu\kappa_{22})w_{3,1} = I_0\ddot{u}_1 + I_1\ddot{w}_1, \tag{4}$$

$$\frac{B}{2}(1+\nu)(u_{1,12} - u_{2,11}) + Bu_{2,\alpha\alpha} + B(\nu\kappa_{11} + \kappa_{22})w_{3,2} = I_0\ddot{u}_2 + I_1\ddot{w}_2. \tag{5}$$

Indicial notation is used throughout this paper. Greek indices range from 1 to 2 and Latin indices take values from 1 to 3. Einstein's summation convention is used unless otherwise indicated. In these equations, w_α represents rotation with respect to the x_1 and x_2 axes, and w_3 represents transverse deflection; $\ddot{w}_\alpha$ denotes angular acceleration with respect to the x_1 and x_2 axes, and $\ddot{w}_3$ represents the transverse acceleration; u_α and $\ddot{u}_\alpha$ represent membrane displacement and acceleration along the x_α axis, respectively; $B = Eh/(1-\nu^2)$, $D = Eh^3/12(1-\nu^2)$, and $C = D(1-\nu)\lambda^2$ are the tension, bending, and shear stiffness of the