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Green's function for frequency analysis of thin annular plates with nonlinear variable thickness

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ABSTRACT

This study considers the free vibration analysis of homogeneous and isotropic annular thin plates with variable distributions of parameters by using the properties of Green's function and Neumann series. The general forms of Green's function depending on the Poisson ratio and the coefficient of distribution for the plate's flexural rigidity and thickness are obtained in closed-form. The fundamental solutions of differential Euler equations are expanded in the Neumann power series using the method of successive approximation based on the properties of integral equations. This approach allows us to obtain the nonlinear frequency equations as a power series that converges rapidly to exact eigenvalues for different power index values and Poisson ratio values. The Neumann power series can then be used to solve the boundary value problem for the free vibration of circular and annular plates with discrete elements, such as an additional mass or ring elastic support. Numerical solutions of the characteristic equations are presented for annular plates with constant and hyperbolic varying thickness, as well as different boundary conditions. The results obtained are compared with selected results from previous studies.

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1. Introduction

The study of the vibration of a thin annular plate is a basic area of structural mechanics because it has many applications in civil and mechanical engineering. Annular plates with constant and variable thickness are the most critical structural elements in high-speed rotating engineering systems. The natural frequencies of annular plates have been studied extensively for more than a century because destruction may occur if the frequency of the external load matches the natural frequency of the plate. In addition, the influence of the distribution of flexural rigidity and mass on the dynamic behavior of plates has been studied frequently because understanding the distribution of the plate's parameters allows us to stabilize or increase (decrease) the frequency of plates. The dynamic characteristics obtained allow us to shape the dynamic behavior of structural elements.

The free vibration of annular plates has received much attention [1–4] and several books [5–7] are excellent sources of information regarding the methods used for the free vibration analysis of circular and annular plates with constant and variable thickness.

Free vibration analysis has been performed using various weighting functions such as the Ritz method, Galerkin method, and finite element method [7]. Kim and Dickinson [8] studied the lateral vibration of a thin annular plate subject to certain complex effects. Liu and Chen [9] analyzed the axisymmetric vibration of circular and annular plates using a simple finite element method. Wang et al. [10] studied the free vibration of circular annular plates with non-uniform thickness using the differential quadrature

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method. Chen [11] studied the axisymmetric vibration of circular and annular plates with arbitrarily varying thickness. Vera et al. [12] investigated the transverse vibration of a free circular annular plate. Selmane and Lakis [13] studied the natural frequencies of the transverse vibrations of non-uniform circular and annular plates using a hybrid method based on plate theory and finite elements. Liu et al. [14] considered the effects of satisfying the stress boundary conditions during the axisymmetric vibration analysis of circular and annular plates. Wang and Wang [15] analyzed the fundamental frequencies of annular plates with a small core. Duan et al. [16] investigated the free vibration of a class of non-uniform annular plates using a generalized hypergeometric function. Rokni et al. [17] studied the free vibration of circular annular plates with variable thickness and different boundary conditions at both edges. Liang et al. [18] analyzed the natural frequencies of circular annular plates with variable thickness using the limited finite element method. Zhou et al. [19] analyzed the natural vibration of thin circular and annular plates using a Hamiltonian approach. Wang [20] studied the vibration modes of concentrically supported free circular and annular plates. Rao and Rao [21] analyzed the free vibration of annular plates where both edges were elastically restrained and resting on a Winkler foundation.

Conway [22] analyzed the basic natural frequency of the axisymmetric vibrations of circular plates with variable thickness and a clamped edge when the flexural rigidity varied with the radius according to a power law, where the characteristic equations were obtained using special Bessel functions for particular cases when the plate thickness changed in a linear and parabolic manner. Several combinations of power index m and Poisson ratios were selected according to the formula: $\nu = (2m-3)/9$, which led to exact solutions. These solutions have limited practical applications. Jaroszewicz et al. [23] considered the same problem by analyzing the basic frequency of the axisymmetric vibrations of circular plates with variable thickness and a clamped edge for different combinations of the parameter m and Poisson ratios according to the formula: $\nu = m^{-1}$. The obtained solutions [23] have limited practical value.

Similarly, the boundary value problem was considered for thin annular plates by Duan et al. [16], who expressed the vibration solutions for axisymmetric power law rigidity in terms of generalized hypergeometric functions. These functions are not readily available and their evaluation requires infinite series summations, which are tedious.

In the present study, we employ Green's functions and the Neumann power series to solve the natural vibration of annular thin plates with constant and hyperbolic varying thickness ($m \le 0$). The general forms of Green's function are obtained for different values of the power index m and Poisson ratio. The fundamental solutions expanded in the Neumann power series allow us to obtain nonlinear characteristic equations that converge rapidly to exact eigenvalues for all combinations of the power index m and Poisson ratio. The Neumann power series obtained allows us to solve the boundary value problem for the free vibration by considering the classes [5,22,23] of circular and annular plates with discrete elements, such as additional mass and ring elastic supports.

We present the ratios of the basic frequencies of hyperbolic and uniform annular plates according to different values of the core radius for various boundary conditions. The numerical results obtained for uniform annular plates are in good agreement with selected previously reported results. The numerical results for this class of non-uniform annular plates have not been reported previously, but the results obtained agree with the physical properties of these types of plates with variable thickness.

2. Statement of the problem

We consider an isotropic, homogeneous annular thin plate with variable thickness $h(r,m) = h_R r^{m/3}$ and flexural rigidity $D(r,m) = D_R r^m$ under cylindrical coordinates (r, θ, z), where the z-axis is along the longitudinal direction. $D_R = Eh_R^3/12(1-\nu^2)$ and h_R are the flexural rigidity and thickness of annular plate at the outer edge ($r = R$), respectively. The geometry and coordinate system for the considered plate are shown in Fig. 1. The partial differential equation for the free vibration of thin annular (circular) plates has the following form

$$D(r,m)\frac{\partial}{\partial r}\left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right) + \frac{\partial D(r,m)}{\partial r}\left(\frac{\partial^2 W}{\partial r^2} + \frac{\nu}{r}\frac{\partial W}{\partial r}\right) + \frac{1}{r}\int_0^r \rho h(r,m)\frac{\partial^2 W}{\partial t^2} r dr = 0, \tag{1}$$

where ρ is the mass density, r is the radial coordinate, and $W(r, \Theta, t)$ is the small deflection compared with the thickness h of the plate.

The axisymmetric deflection of an annular plate may be expressed as follows

$$W(r,t) = w(r)e^{i\omega t}, \tag{2}$$

where $w(r)$ is the radial mode function, ω is the natural frequency, and $i^2 = -1$. By substituting Eq. (2) into Eq. (1) and using the dimensionless coordinate $\xi = r/R$, the governing differential equation of the annular (circular) plate becomes:

$$L_m(w) - \lambda^2 \xi^{-\frac{2m}{3}} w = 0, \tag{3}$$

where $L_m(w)$ is the differential operator defined by

$$L_m(w) \equiv \frac{d^4 w}{d\xi^4} + \frac{2(m+1)}{\xi}\frac{d^3 w}{d\xi^3} + \frac{(m^2 + m + \nu m - 1)}{\xi^2}\frac{d^2 w}{d\xi^2} + \frac{(m^2\nu - m\nu - m + 1)}{\xi^3}\frac{dw}{d\xi}, \tag{4}$$

and the dimensionless frequency of vibration λ is given by

$$\lambda = \omega R^{m/3}\sqrt{\rho h_R/D_R}\,. \tag{5}$$

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