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# Bending and twisting friction models in soft-sphere discrete element simulations for static and dynamic problems

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ABSTRACT

In soft-sphere discrete element models of granular flow, particles may interact in a variety of ways including interactions normal to points of contact and interactions tangential to points of contact such as sliding, rolling, bending and twisting. In the majority of models normal and sliding modes are used. Rolling friction is sometimes reported but incorporation of bending and twisting effects is less common. In this paper it is shown that the precise mathematical nature of bending and twisting models in soft-sphere simulations can have significant effects on model predictions, especially for the case of dynamic granular flow problems.


## 1. Introduction

This paper arises out of industrial research concerning the development of new material distribution prediction methods for the Port Talbot Works of TATA steel. The industrial focus of the work is a better understanding of the charging of raw material (e.g. coke, iron ore and sinter) collectively known as 'burden' into blast furnaces [1]. Kurunov identified that burden charging in the blast furnace affects furnace productivity and that the choice of charging system can improve furnace productivity by up to 7% and reduce coke usage by up to 7.6% [2]. However, to do so requires an understanding of dynamic 3D loading patterns.

The blast furnace is a hostile environment which makes in-process monitoring extremely difficult. Realistic simulation of the dynamic granular flows within the blast furnace is therefore highly desirable. Discrete Element Method (DEM) has been used by several researchers to model blast furnaces although these models often assume radial symmetry and are not true scale simulations [3–5]. While true scale simulation has been reported [1], the details of the tangential interactions between particles can have a significant effect on model predictions. This is especially true for the largely dynamic case of blast furnace charging where continuous granular flow of material is important rather than the simpler case of static pile formation of granular materials. This prompted the investigation described here. In this paper the effects of tangential forces in soft-sphere DEM models are specifically investigated for two test cases. The case of static pile build up in a previously reported 'ledge test' example is considered first. Then a second dynamic case of a rotating drum containing granular material is investigated.

This paper limits itself to descriptions of the common frictional models used in DEM simulations. The usual differentiations between various types of friction are used namely shear and rolling, where rolling can be decomposed into bending and twisting (where some authors refer to twisting as torsion).

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### 1.1. Shear friction

Sliding friction, $F_s$, is handled using the widely used linear damped spring in series with a sliding friction element which can be summarised as,

$$F_s = \min\left(k_s\delta_s + \nu_s\dot{s},\ \mu_s|F_n|\right) - \hat{s}, \tag{1}$$

$$F_n = k_n\delta\hat{n}, \tag{2}$$

$$\nu_s = 2\sqrt{K_s\,\frac{m_i m_j}{m_i + m_j}} - \frac{\log(1/\eta_n)}{\sqrt{\pi^2 + \log(1/\eta_n)^2}}, \tag{3}$$

where $F_n$ is the normal contact force, $k_n$ is spring strength, $\delta$ is overlap between spheres, $\hat{n}$ is the direction of the normal, $\mu_s$ is the coefficient of sliding resistance, $k_s$ is a spring stiffness, $\delta_s$ is a contact overlap, $\nu_s$ is a dampening term, $\eta_n$ is the coefficient of restitution, $m$ is the mass of a particle, $\hat{s}$ is the surface tangent vector and $\dot{s}$ is relative velocity between a particle and another particle or object [6]. Here the tangential stiffness is constant for the no-slip condition, shear tractions are singular at the edges of the contact region, and there is a non-linear stiffness for a constant normal load and a monotonically increasing tangential load.

### 1.2. Rolling friction

Rolling friction is a resistive force that slows down the motion of a rolling particle and is typically a combination of several frictional forces at the point of contact between the rolling particle and another particle or surface. In reported DEM models there are several ways of incorporating rolling friction effects. Zhou et al. [7] describe conventional treatments of rolling friction where the friction may be either (i) independent of the angular velocity or (ii) directly proportional to the relative angular velocity of two particles in contact.

### 1.3. Case (i): direction-constant torque model

The direction of the rolling frictional torque always *opposes* the relative rotation and is proportional to the normal contact force. This is a typical direction-constant torque model. In a 2D for model (i) the torque between two contacting discs, *i* and *j*, can be expressed using a normalised relative angular velocity as,

$$T_r = -\frac{\omega_{rel}}{|\omega_{rel}|}\mu_r R_r F_n, \tag{4}$$

$$\omega_{rel} = \omega_i - \omega_j, \tag{5}$$

where $\mu_r$ is the coefficient of rolling resistance, $\omega_i$ and $\omega_j$ are the angular velocities of disks i and j respectively, $\omega_{rel}$ is the relative angular velocity between them and $R_r$ is the so-called 'rolling radius' given by,

$$R_r = r_i r_j/(r_i + r_j), \tag{6}$$

where $r_i$ and $r_j$ are the radii of contacting particles *i* and *j*.

### 1.4. Case (ii): viscous model

Rolling frictional torque is proportional to the translational velocity arising from the relative angular velocity at a contact point between two particles as,

$$T_r = -\mu_r R_r F_n(\omega_i r_i - \omega_j r_j). \tag{7}$$

This is a typical viscous model. 3D numerical results on sand pile simulations showed that treatment (i) gave better results than treatment (ii). Zhou et al. [8] subsequently assessed this methodology by comparing to experimental data for mono-sized spheres. Combinations of approaches (i) and (ii) have also been used [9] where the rolling frictional torque contains both viscous and slider effects represented as,

$$T_r = \min\left(-\mu_r|F_n|, -\mu_r|\omega_{rel}|\right)\left(\frac{\omega_{rel}}{|\omega_{rel}|}\right), \tag{8}$$

where $\omega_{rel}$ is the vector of the *relative* tangential rotation of particles *i* and *j*.

Approaches (i) and (ii) above, plus a third approach (case (iii) below) using an elastic–plastic spring-dashpot model, were assessed in 2D by Ai et al. [10].