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# A note on a two variable block replacement policy for a system subject to non-homogeneous pure birth shocks

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ABSTRACT

This paper considers an operating system subject to shocks occurring as a non-homogeneous pure birth process (NHPBP). A shock can cause two types of system failures: a type-I failure (minor failure) is rectified by a general repair, whereas a type-II failure (catastrophic failure) is removed by an unplanned replacement or remains inactive until the next planned replacement. The probabilities of these two types of failures depend on the number of shocks since the last replacement. We consider a modified block replacement policy (BRP) policy with a random repair cost as follows: first, an operating system is preventively replaced with a new one at times $iT$ $(I = 1,2,\ldots)$ regardless its failure history. If the system fails in $[(i-1)T, (i-1)T+T_0)$, it is either replaced by a new one or generally repaired, and if fails in $[(i-1)T+T_0, iT)$, it is either generally repair (minor failure) or remains inactive (catastrophic failure) until the next planned replacement. Thus the policy is called a two variable policy with a fixed replacement interval and a threshold for the system age. The expected cost rate is derived by using the renewal reward theory. The two-variable cost function is transformed into a one variable function and the optimal policy is analyzed.



## 1. Introduction

Consider a system subject to shocks which cause the system to break down. In such a situation, it is important to determine a preventive replacement policy that minimizes the maintenance cost of the system. We focus on a reasonable block replacement policy (BRP) first introduced by Barlow and Proschan [5]. Under a classical BRP, an operating system is replaced with a new one at planned times $iT, i = 1, 2, \ldots,$ and at unplanned failures. This policy is commonly used when there are a large number of identical systems in service as it is easier to plan and implement. The main drawback of BRP is that it can be costly because sometimes fairly new systems are also replaced at planned replacement times. To overcome this weakness, various modified BRPs have been suggested. These include BRP with minimal repair at failure [4], BRP with used system at failed replacement [34], BRP with two variables that the system is replaced or remain inactive depending on the time to the planned replacement at the failure instant [20], BRP with minimal repair limit [18], and BRP with two variables and general random minimal repair cost [25].

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**Nomenclature**

| | |
|---|---|
| $\{N(t): t \geq 0\}$ | non-homogeneous pure birth process (NHPBP) with intensity function $\{\lambda_k(t), k = 0, 1, 2, \ldots\}$ |
| $\Lambda_k(t)$ | mean value function of $\{N(t): t \geq 0\}$; $\Lambda_k(t) = \int_0^t \lambda_k(x)\mathrm{d}x$ |
| $P_k(t)$ | transition probability at time $t$ given $N(0) = 0$ |
| $T$ | time interval of a planned replacement |
| $T_0$ | time point at which the system is replaced or remains inactive when a type-II failure occurred in a replacement cycle; $0 \leq T_0 \leq T < \infty$ |
| $M$ | number of shocks proceeding the first type-II failure |
| $p_k, \bar{P}_k$ | probability mass function, survival function of $M$; $\bar{P}_k = P(M > k)$=Pr{first $k$ shocks are type-I failure}, $p_k = P(M = k) = \bar{P}_{k-1} - \bar{P}_k = \bar{P}_{k-1}(1 - \bar{P}_k/\bar{P}_{k-1})$ |
| $\{\bar{P}_k\}$ | a sequence of $\bar{P}_k$ |
| $q_k$ | Pr{a type-I failure occurred when shock $k$ arrives}; $q_k = \bar{P}_k/\bar{P}_{k-1}$ |
| $\theta_k$ | Pr{a type-II failure occurred when shock $k$ arrives}; $\theta_k = 1 - q_k$ |
| $R_1$ | cost of an unplanned replacement due to type-II failure |
| $R_2$ | cost of a planned replacement due to age $T$ |
| $R_3$ | cost rate during the inactive period |
| $g(C(t), c_k(t))$ | cost of the $k$th general repair at age $t$, where $C(t)$ is the age-dependent random part and $c_k(t)$ is the deterministic part which depends on the age part and the number of the general repairs |
| $n_k(t)$ | expected value of $g(C(t), c_k(t))$; $n_k(t) = E_{C(t)}[g(C(t), c_k(t))]$ |
| $Y$ | time interval between two successive unplanned replacements |
| $h(t), \bar{H}(t)$ | probability density function, survival function of $Y$ |
| $V(t)$ | renewal function associated with $\bar{H}$ |
| $K(T; T_0, T)$ | total maintenance cost over $[0, T)$ with expectation $E[K(T; T_0, T)]$ |
| $K([0, T_0); T_0, T)$ | total maintenance cost over $[0, T_0)$ with expectation $B(T_0)$ |
| $K([T_0, T); T_0, T)$ | total maintenance cost over $[T_0, T)$ with expectation $O(T_0, T)$ |
| $J(T_0, T)$ | expected long-run cost per unit time of a replacement cycle |
| $S_k$ | arrival time of the $k$th shock with $S_0 = 0$ |
| $Y(T_0)$ | excess or residual life of the system in use at $T_0$ |
| $T*, T_0^*$ | optimal solutions that minimize $J(T_0, T)$ |

Shock models have been extensively studied in reliability theory. Esary et al. [14] assumed that shocks occur according to a homogeneous Poisson process (stationary case). A-Hameed and Proschan [1] treated the case with shocks following a non-homogeneous Poisson process (NHPP). NHPP is commonly used in maintenance and applied probability literature. Some recent applications with NHPP shock model were reported in the work by Chang et al. [11], Sheu et al. [28], and Sheu et al. [30]. However, the NHPP shock process only depends on the system's age and cannot capture the effect of accumulated number of failures which can affect the system's reliability significantly. To overcome this NHPP limitation, one can model the shocks as a non-homogeneous pure birth process (NHPBP) which allows the shock process to depend on both the number of failures and the system age. A-Hameed and Proschan [2] presented a pure birth shock model with a non-stationary Markov process as follows: given $k$ shocks have occurred in $[0, t]$, the probability of a shock occurring in $(t, t + \Delta)$ is $\lambda_k\lambda(t)\Delta + o(\Delta)$. Sheu et al. [29] considered a NHPBP with intensity $\{\lambda_k(t), k = 0, 1, 2, \ldots\}$, which generalized the model in A-Hameed and Proschan [2]. Such a NHPBP is more appropriate for a situation where the number of repairable failures indicates the system condition. Other applications of NHPBP can be found in insurance, epidemiology, and load-sharing models [3, 17, 23, 29, 31, 32].

In this paper, a modified two-variable BRP is analyzed for a system subject to NHPBP shocks. We extend the model in Sheu [25] to a NHPBP shock model with a general repair cost structure. In fact, such a policy can be effective for operating and maintaining a group of identical systems in practice, such as digital computers and other high-tech electronic devices. For example, computers in a student lab at a university are replaced at the beginning of every fixed number of years due to the fast technology development (for both hardware and software). During the time between any two consecutive replacements, if a minor failure occurs, a general repair is performed to ensure that the computers are functional for classes. However, if a catastrophic failure occurs, the decision to replace the system or remain inactive until the next planned replacement is often depending on the consideration of some important time point such as warranty. Such a system maintenance problem fits our model. The main objective of this note is to propose a modified BRP with two variables and NHPBP shocks. We also discuss on optimizing the policy parameters for the proposed model.

The rest of this paper is organized as follows. Section 2 describes the system and the NHPBP shock process. Section 3 presents the two-variable BRP policy model and develops the expected cost functions. Section 4 focuses on the optimization of the proposed policy. Section 5 shows that several maintenance models are special cases of our model. Finally, Section 6 concludes.