Applied Mathematical Modelling 40 (2016) 3783–3800

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An effective fracture analysis method based on the virtual crack closure-integral technique implemented in CS-FEM

W. Zeng[a,b,*], G.R. Liu[b], C. Jiang[b,c], X.W. Dong[b,d], H.D. Chen[b,e], Y. Bao[b,f], Y. Jiang[g]

[a] CEAS-Biomedical Engineering (BME), University of Cincinnati, 2851 Woodside Dr., Cincinnati OH 45221, USA
[b] CEAS-School of Aerospace Systems, University of Cincinnati, 2851 Woodside Dr., Cincinnati, OH 45221, USA
[c] State Key Laboratory of Advanced Technology of Design and Manufacturing for Vehicle Body, Hunan University, 410082, China
[d] College of mechanical and electronic engineering, China University of Petroleum, 66 Changjiang Rd, Huangdao District, Qingdao 257061, China
[e] State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei 230026, China
[f] Department of Geotechnical Engineering, College of Civil Engineering, Tongji University, Shanghai 200092, China
[g] Hefei General Machinery Research Institute, 888 West Changjiang Rd., Hefei 230031, China

ARTICLE INFO

Article history:
Received 16 July 2015
Revised 24 September 2015
Accepted 2 November 2015
Available online 6 November 2015

Keywords:
Fracture mechanics
Virtual crack closure integral method (VCCM)
Strain smoothing techniques
Cell-based smoothed finite element (CS-FEM)
Mixed-mode fracture
Crack propagation

ABSTRACT

In this work, the virtual crack closure integral technique (VCCT) is formulated in the framework of cell-based smoothed finite element (CS-FEM) for evaluating stress intensity factors and for modeling the crack propagation in solids. In the present CS-FEM, the strain smoothing technique is operated over the smoothing domains which are constructed based on elements, and each element is further subdivided into several smoothing cells. The smoothed strain is then obtained by a boundary integral along the boundaries of the smoothing cells. Only shape function itself is involved in computing the strains and no derivatives of the shape functions or coordinate transformation is required for the computation of the discretized stiffness matrix, and thus ideal for fracture mechanics problems to evaluate the stress intensity factors, we utilize the one-step-analysis approach of the VCCT based on the assumption that an infinitesimal perturbation of crack-tip location shall not significantly affect the stress/displacement field. The significant feature of the present CS-FEM method equipped with VCCT is that it requires no domain integration in the analysis of fracture mechanics problems. Several numerical examples are presented to validate the effectiveness of the present method. It uses only the information for displacement openings behind the crack-tip and the nodal forces at the crack-tip for stress intensity factor evaluation. The comparison study has shown that is as accuracy as the FEM-Q4 that need domain integrations for both stiffness matrix computation and interaction integral methods for stress intensity factor evaluation. The present method is further used to successfully predict the crack growth trajectory with excellent agreement between numerical results and the experimental observations.

1. Introduction

In modern fracture mechanics, the well-known Irwin's work [1] uses the Westergaard approach and showed that, for elastic materials, the stresses and displacements near the crack-tip could be described by a single constant*K*, which is related to the strain energy release rate *G* [2]. Likewise, the Eshelby–Cherepanov–Rice *J*-integral [3–6] has then provided a way to calculate

* Corresponding author. Tel.: +1 513 556 2661.
E-mail address: zengwe@mail.uc.edu, zeng.work@gmail.com (W. Zeng).

http://dx.doi.org/10.1016/j.apm.2015.11.001
S0307-904X(15)00706-4/

the strain energy release rate, or work/energy per unit fracture surface area. In general, almost all the fracture properties of a solid with elastic material can be characterized using a couple of parameters extracted from the near-tip stress and displacement fields. For example, the stress intensity factors (SIF's) K defines the amplitude of the crack tip singularity. Since closed-form analytical solutions for these parameters are only available for some simple problems, numerical modeling techniques such as finite element methods (FEM) and boundary element methods (BEM) are utilized almost exclusively.

To predict facture parameters such as the stress intensity factors, a few classical approaches have been developed including the displacement correlation method (DCM, i.e., displacement extrapolation method [7–10]), domain integral (DI) methods (e.g., interaction integral (I-integral) method [11,12]), stiffness derivative method or virtual crack extension approach (VCE) [13,14], virtual crack closure technique (VCCT) [15,16], etc. In general, most of these methods could be classified into two categories: the point matching approaches (or direct approaches) and the energy based approaches. The inferring of SIFs by the first category of approaches uses stress or displacement fields near the crack-tip directly and the accuracy depends on the nodal displacement or stress directly, which is hence sensitive to associated mesh in FE computations. On the other hand, the energy based approaches evaluate the energy release rates in the body and relate G to SIF's. The energy based approaches, in general, are applicable to elastic as well as elastic-plastic materials and they perform insensitive to mesh quality at the vicinity of the crack tip/front. Nonetheless, the point matching approach would be usually more simple and easy to implement in FE programming.

Before evaluation of the stress intensity factors, the energy based approaches need firstly to compute the strain energy release rate based on results obtained from finite element analysis (FEA). There are a variety of methods to compute the energy release rates, among which three popular ways are usually used: J-integral method (or I-integral), virtual crack extension method (VCEM) and virtual crack closure integral method/technique (VCCM or VCCT). The VCEM computes the energy difference during two finite element analyses when a crack extends for a small amount of increment. While the energy in VCCM, which is required to close the crack for one finite element length, is calculated via multiplying the nodal reactions and the opening displacements. This approach was first proposed by Rybicki and Kanninen [15] for 2D crack problems and was later improved by Raju et al. [17,18]. It was also extended for 3D analysis of crack problems by Shivakumar et al. [16]. When inferring the energy release rate from FEA results, the VCCT has several advantages, such as simplicities in computation which involves only the nodal reaction forces at the crack tip/front and opening displacements just behind the tip/front, and convenience of mode separation when appropriate vector components are utilized in computation of components of the energy release rates.

Over the past two decades, the VCCT has been developed and extended to various aspects. For example, Refs. [19,20] used VCCT to treat delamination between the face sheet and the core material of sandwich structures. Xian et al. [21–23] proposed the VCCT formulations for kinking cracks and for a moving delamination front of arbitrary shape. Based on FEA results, Sun and Qi an [24] computed the strain energy release rates for interfacial cracks between two isotropic materials. Laski [25] provided the implementation of VCCT in engineering FE calculations and introduced the general conditions of applying the VCCT in conjunction with commercial programs (MSC Patran). In order to extend to 3D problems, Okada et al. [26,27] developed VCCT schemes to compute the energy release rates and stress intensity factors for both linear and quadratic tetrahedral finite elements. Whitcomb [28] computed the strain release rate distributions along a post-buckled embedded delamination including the contact effects. Fawaz [29] performed the sensitivity study of mesh pattern on the accuracy of the calculation of strain energy release rates. In Ref. [30], Krueger presented an overview of historical development of VCCT and a discussion of its applications in engineering. For more details of recent development and applications related to VCCT, we can refer to this review and the references cited therein.

Though the FEM has become the most popular and powerful numerical tool for practical problems in engineering and science including fracture mechanics, it does not mean that it is perfect and no room for improvement. For example, FEM exists the overestimation of stiffness of solid and structures, which may result in locking behavior and inaccuracy in stress solutions [31]. By incorporating the strain smoothing technique [32] into finite element method (FEM), Liu et al. have formulated a series of smoothed finite element methods (S-FEMs) containing cell-based S-FEM (CS-FEM) [33–35], node-based S-FEM [36,37], edge-based S-FEM [38–40], face-based S-FEM [41,42] and combinations of these techniques [43–45]. Several theoretical aspects of the S-FEM models have been provided in Refs. [46–49]. Owing to the strain smoothing operations, the "over-stiff" feature of the standard FEM can be reduced or alleviated and hence the accuracy of both primal and dual quantities can be improved significantly [50]. Moreover, S-FEM does not require the shape function derivatives and S-FEM models developed in elasticity are insensitivity to mesh distortion because the absence of isoparametric mapping [40]. Each of these smoothed FEM has different properties and has been applied to various types of practical mechanics problems. Due to its versatility, the class of S-FEMs has been becoming a simple and effective numerical tool for solving numerous physical problems.

In this work, the VCCT will be formulated and implemented based on the CS-FEM framework. The elements in an FE base mesh will be further subdivided into several smoothing cells (SCs) (e.g., 4 SCs). The Galerkin weak form is used as in FEM, but the strain field at any point in an element is defined as a weighted spatial average over the element. Through such smoothing operation, the obtained smoothed strains will be then used for computing the stiffness matrix. If piecewise-constant weight functions are adopted, area integrations over the domain of cell in the weak form become contour integration along the boundaries of the smoothing cells. As a result, only shape functions themselves (not the derivatives) will be involved in computing the field gradients to form the stiffness matrix. Numerical studies [33,51–56] have demonstrated that CS-FEM shows some interesting properties compared to the standard FEM using 4-node isoparametric elements. For examples, (1) CS-FEM can obtain better results than those of FEM in both displacement and energy because of the softening effect; (2) less strict mesh quality requirements as no coordinate transformation or mapping is involved in CS-FEM (e.g., Abaqus will not run a job with a Jacobian below 0, and requires the Jacobian be great than 0.2 for a solid element and 0.3 for a shell element); (3) construction of shape