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Analysis of shear banding phenomena in non-isothermal flow of fluids governed by the diffusive Johnson–Segalman model

I.E. Ireka[a], T. Chinyoka[a,b,*]

[a] Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, South Africa
[b] Center for Research in Computational & Applied Mechanics, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa

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ABSTRACT

This article investigates phenomena associated with shear banded flows of viscoelastic fluids in both pipe flow and coaxial annular flow. We in particular computationally analyze the effects of stress diffusion, non-isothermal flow conditions and annular gap size on the shear stress path selection (and the selected shear stress value) in shear banded flow of viscoelastic fluids described by the non-local Diffusive Johnson–Segalman (DJS) constitutive model. The DJS model is one of such constitutive models which allows us for a non-zero inter-facial layer between regions of high and low shear rates within the flow field. Temperature effects due to polymer orientation changes, entropic effects (resulting from stress work), conduction heat transfer, Arrhenius chemical kinetics as well as the influence of slip and related frictional heating on the wall are all accounted for. The time dependent model formulated to capture gap effect in the axial annular flow problem (under certain values of the relevant parameters), reduces to the system of equations governing a pipe flow setup. Semi-implicit finite difference methods are employed for the solution process of the coupled nonlinear time dependent partial differential equations governing the flow problem. We discuss with graphical representations the effect of temperature, stress diffusion, wall slip, suction/injection and annular gap size on the related shear rate path selection phenomena for certain material parameter values.

1. Introduction

Industrial relevance of macromolecular fluids and their commercial viability have led to increased research activity and interest in the dynamics of these complex fluids. One such flow problem which has attracted significant theoretical and experimental research activity is the flow of viscoelastic fluids through annular duct. This is mainly due to its vast application in several industrial processes such as oil extraction and drilling, plastic extrusion, journal bearings, wire coating, annular heat exchangers in food processing etc. The theoretical analyses have been conducted for various fluid models capable of predicting, to a certain extent, the complex dynamics of non-Newtonian fluids under shear. Fluid models such as the Bingham model, Power law model, Oldroyd-B model, Johnson–Segalman model, Phan–Thien–Tanner model and the Geisekus model, have all been used by various authors, see for example [1–8], to investigate flow patterns/profiles or volumetric flow rate in annular flow of non-Newtonian fluids under certain physical conditions.

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[*] Corresponding address at: Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, South Africa. Tel.: +27729749532.

E-mail address: tchinyok@vt.edu (T. Chinyoka).

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Due to their macromolecular and viscoelastic nature, polymeric fluids when subjected to an imposed shear stress (such that a critical relaxation time is exceeded) exhibit certain flow instabilities [9]. These instabilities emerge within the flow as shear bands or fluid layers flowing with different internal configuration and apparent viscosities, coexisting and separated along the normal to the flow direction [13]. This phenomena of “gradient banding” in flow of complex fluids has been widely investigated both theoretically and experimentally, see for example [13,14,18,20]. For instance, the experiments of Divoux et al. [23] on flow of yield stress fluids, show that transient shear banding can occur in the flow even before steady state is reached. The onset of shear bands in simple shear flow of complex fluids can be attributed to multiple weak steady state solutions of the fluid’s constitutive relation resulting from a non-monotonic shear stress - shear rate relationship, with the region of negative slope representing an unstable regime within the flow [21]. These non-monotonic constitutive relations have been observed to possess inherent history-dependence which could be resolved by introducing diffusion terms [22]. Several fluid models exhibiting this phenomena, while offering good comparison with experimental data, include Giesekus model [10], Rolie–Poly model [11], the VCM model [12] and Johnson–Segalman (JS) model [24] amongst others. Relevant flows of fluids described by these models have been extensively investigated under both isothermal conditions, see for example [13–17,19] (and the references therein) and non-isothermal conditions, [25,26].

Studies have demonstrated that in order to ensure a mechanism to select a unique shear stress at which shear banding will occur, one will need to incorporate non-local terms into the equations governing the stresses (i.e. those formulated in [10,11] and [24]) see for example [13,19]. In this study, we focus on the Johnson–Segalman model which in its original form, has no mechanism to ensure selection of the unique shear stress at which shear banding will occur. Experimentally, however, the selected shear stress (and hence shear rate path) is usually observed to be independent of flow history and initial conditions [13]. With the addition of diffusion term in the JS model, the resulting Diffusive Johnson–Segalman (DJS) model is capable of predicting the experimentally observed unique total stress at which shear banding occurs. This model has been proposed either with polymer stress diffusion [27] or shear rate diffusion [28]. It is important to note that the total stress corresponds to the stress at which the interface between shear bands is stationary, and the diffusion terms which determines the non-zero inter-facial width, describes nonlocal relaxation of the viscoelastic stresses necessary for describing strongly inhomogeneous flow profiles. For a detailed explanation on stress selection phenomena in shear banded flows, we refer the reader to the comprehensive review paper of Olmsted [13].

The small diffusion coefficient introduced in this investigation makes a significant difference in the behavior of the shear stress - shear rate curves around the regions of non-monotonicity. Away from the regions of non-monotonicity, the shear stress - shear rate curves are otherwise unaffected by the addition of stress (or shear rate) diffusion. Physically, the stress diffusion terms contribute to particle diffusion across inter-facial layers between the bands and hydrodynamic interactions within each band. However, recent study has shown that this phenomena can be accounted for more explicitly without stress diffusion [17].

In modeling flow of viscoelastic fluids, it is important to accommodate their energy storing ability. For instance, in the flow of polymeric fluids, effects such as (i) energetic effects due to polymer orientation changes, (ii) entropic effects resulting from stress work and (iii) conduction heat transfer effects, should be accounted for in the models, [8,29–32]. In fact, it has been shown that non-isothermal wall conditions affects the magnitude of wall shear stress [25,33]. Another important phenomena which significantly contributes to instabilities in flow of polymeric fluids is wall slip (see [9,25,34] and the references therein). Investigations on the interplay between temperature effects, wall slip and shear banding reveal an interdependence of non-isothermal conditions, wall slip and shear banding [25].

The current investigation is motivated by relevant applications of annular flow of viscoelastic fluids as well as the experimental results of Olmsted [18] where it was established that confinement and boundary conditions influence nonlocal effects in flows of worm-like micellar, and the work of Cromer et al. [16], where pressure driven flow of worm-like micellar solutions through rectilinear micro-channels was investigated using the VCM model. Our study incorporates polymer stress diffusion into the stress constitutive relations of Johnson–Segalman model, as suggested by [27], instead of the shear rate diffusion type of [28] which was also adopted in [18]. However, introducing stress diffusion warrants that the stress relations are solved subject to a new set of boundary conditions; i.e. the stress equations becomes parabolic instead of hyperbolic. We formulate the model for the annular flow problem and include a gap parameter χ, which when set to 1, reduces the model to the set of equations governing a pipe flow. For the pipe flow problem, we assume the stresses to be zero at the axis (i.e. at $r = 0$, which corresponds to a steady state solution of the total stress from the momentum equations as $r \to 0$ at constant axial pressure gradient), and then take the gradient of the stresses at the wall along the outward normal to be zero, as in [13]. For the annular flow problem, we also take the gradient of the stresses at the walls, along the outward normal to be zero. The emerging unsteady nonlinear coupled partial differential equations are solved via semi implicit finite difference techniques. Most importantly, our aim in this study is to investigate possible effects of annular gap size, polymer stress diffusion, fluid injection through the inner pipe, non-isothermal and wall slip conditions on the selected shear stress for a polymeric fluid described by the DJS constitutive model. We discuss with graphical illustrations these influence on the flow profiles.

2. Model formulation

2.1. Model assumptions

To formulate the mathematical model describing the physical system under study, we assume that

- The flow is a startup pressure driven flow in a concentric annular pipe.