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A novel mathematical formulation for predicting symmetric passive bipedal walking motion with unbalanced masses

P. Mahmoodi, R.S. Ransing*, M.I. Friswell

Zienkiewicz Centre for Computational Engineering, College of Engineering, Swansea University, Swansea, UK

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ABSTRACT

Commercial prosthetic feet weigh about 25% of their equivalent physiological counterparts. The human body is able to overcome the walking asymmetry resulting from this mass imbalance by exerting more energy. It is hypothesised that the passive walking dynamics coupled with roll-over shapes has potential to suggest energy efficient walking solutions. A two link passive walking kinematic model has been proposed to study the gait pattern with unbalanced leg masses. An optimal roll-over shape for the prosthetic foot that minimises the asymmetry in the inter-leg angle and the step period is determined. The proposed mathematical formulation provides insights into the variation of step length and inter-leg angle with respect to the position and location of the centres for mass of both prosthetic and physiological legs.

1. Introduction

A prosthetic leg is an artificial limb and effectively is a dead weight. It is not supported by human muscles and the associated nervous system. Some of the commercially available prosthetic feet weigh only a quarter of the corresponding weight of the human leg. This paper investigates whether this mass imbalance has an influence on the kinematics of the gait. This is important as the human desire to maintain a symmetric walking gait irrespective of imbalances may increase energy consumption. Many gait descriptors have been presented to study the asymmetry of walking with a prosthetic foot [1,2]. However, they did not include roll-over shapes. The roll-over shape is the locus of the centre of pressure as the foot rolls over the surface. It is defined in a local co-ordinate system with the y axis aligned to the ankle–knee axis. The roll-over shape has been widely used to understand the knee–ankle–foot kinematics as it is invariant to many gait parameters such as added weight, speed and shoe heel height [3–5]. The ideal roll-over shape for the prosthetic foot may be realised by aligning the prosthetic foot using number of techniques [6]. In this paper walking asymmetry is evaluated using the inter-leg angle and the step period. An example of asymmetric walking is shown in the schematic diagram in Fig. 1. If both physiological and prosthetic feet have different masses but the same roll-over shape then the kinematic model suggests that the walking is asymmetric. This is shown by the shadowed triangle in Fig. 1 describing the asymmetry in terms of the inter-leg angle, as well as the step length.

The proposed mathematical formulation exploits the benefits of passive walking [7–9] and develops an optimal roll-over shape for the prosthetic foot so that the walking is stable and symmetric as shown in Fig. 1(b). The mathematical formulation produces an optimal region for roll-over shape gain values that minimises asymmetric walking with respect to both the inter-leg angle (solid curve in Fig. 1(c)) and the step period (dotted curve in Fig. 1(c)).

* Corresponding author. Tel: +44 1792 295219; fax: +44 1792295676.
E-mail address: r.s.ransing@swansea.ac.uk (R.S. Ransing).

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Nomenclature

Parameters	Description
θ_s	angle of stance leg
θ_{vs}	angle of virtual stance leg
θ_{vsl}	angle of virtual stance lower leg
θ_{ns}	angle of swing leg
ϕ	slope angle
μ	$\frac{m_H}{m_{physiological\ leg}}$
β	$\frac{physiological\ upper\ leg\ length}{physiological\ lower\ leg\ length}$
l_{vs}	length virtual stance leg
a_{vs}	length virtual stance lower leg
m_H	mass of hip joint
m_s	mass of stance leg
m_{ns}	mass of swing leg
υ	$\frac{m_{prosthetic\ leg}}{m_{physiological\ leg}}$
γ	$\frac{prosthetic\ upper\ leg\ length}{physiological\ upper\ leg\ length}$

Gard et al. [10] studied the kinematics of walking motion and measured the trajectory of a mass as it rolls on a circular arc. It was demonstrated that the trajectory can be modelled by a simple inverted pendulum but with a longer virtual leg length extending to a virtual floor beneath the actual floor. However, even simple double pendulum dynamics is not sufficient to model the biped walking motion. The distances between the rolling contact point and the hip and leg masses change during the rolling motion and must be accurately accounted for in the mathematical formulation.

Mahmoodi et al. [11] gave a detailed review of the applicability of the roll-over shape for a biped walking process and of various computational approaches for modelling biped walking motion. They proposed a mathematical formulation to integrate an arbitrary shaped roll-over shape into a passive walking biped model. However, the formulation was limited in using identical masses for both legs. The proposed formulation, as described in Section 2 and Appendix A, is designed to study the effect of the mass imbalance on walking symmetry resulting from using a lighter prosthetic foot. Section 2.2 verifies the accuracy of the proposed model by reproducing the circular trajectory of the hip mass as it rolls on a circular arc. It is observed that the actual roll-over shapes are not symmetric around the ankle joint as the forefoot length is always longer than the hindfoot length. The predicted trajectories are shown. Representative anthropometric and prosthetic foot data is used to demonstrate the passive biped walking model and the results are discussed in Section 3. For the physiological foot a roll-over shape is chosen that is

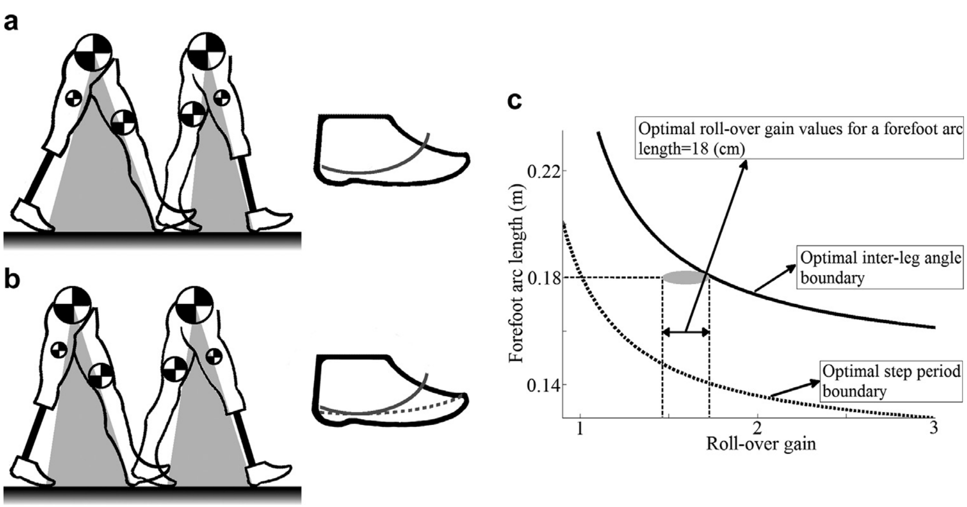

Fig. 1. Walking with identical roll-over shapes (a) and with an optimal roll-over shape (b). The grey region represents the inter-leg angle and the dotted curve shows an optimal roll-over shape. (c) The solid and dotted lines describe symmetric gait with respect to the inter-leg angle and step period respectively. The ellipsoid shows the range of optimal roll-over gain for a forefoot arc length of 18 cm. Refer to the end of Section 3 for further discussion.