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Laminar pipe flow with drag reduction induced by a magnetic field gradient

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ABSTRACT

This work presents analytic and numerical results for a unidirectional flow in a circular capillary of a magnetic fluid. A condition that deviates from the standard equation of equilibrium magnetization due to the vorticity of the flow is considered. We show that the model of a magnetic symmetrical fluid with no-coupling between magnetization and flow leads to velocity profiles very close to the parabolic one. This scenario may change drastically when the fluid magnetization is coupled with the flow vorticity resulting in a nonlinear relation between pressure gradient and flow volume rate and consequently a non-Newtonian behavior. The solutions are developed with the fluid undergoing each a pressure gradient and an applied magnetic field gradient. The dimensionless governing equations reveal an important physical parameter denoted magnetic Reynolds number, which measures the relative importance between hydrodynamic and magnetic forces. The resulting equation is solved numerically and by a regular perturbation method. Explicit expressions for the dimensionless velocity profile, effective viscosity and friction factor as function of the dimensionless magnetic and hydrodynamic physical parameters are presented. Numerical and analytic results are compared and a good agreement is verified for the range where the analytic solution can be used. The studies are applied to explore the emerging propose of drag reducing fluids by applying a magnetic field gradient favorable to the flow direction. We have observed that the application of a controllable magnetic field gradient may be responsible for a meaningful drag reduction in a laminar pipe flow.

1. Introduction

Magnetic fluids, or ferrofluids, are colloidal suspensions of small magnetic particles dispersed in a carrier liquid, water or oil in most cases. These solid particles, typically ferrite and magnetite, with diameters varying from 3 to 15 *nm*, can be treated as permanent nano-magnets in random motion due to Brownian forces, which prevents particles from settling under the action of gravity [1]. The distribution of electric charges or a surfactant layer around each particle ensures the magnetic fluid stabilization, avoiding irreversible aggregates formation. A typical magnetic fluid has about 10^{23} particles per cubic meter, is opaque to the naked eye and the particle volume fraction is about 1% [2,3]. Good review works on magnetic fluids and their applications are also available in the current literature [1,2,4].

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After the pioneering works of [5] and [6], several studies were developed on viscous properties of ferrofluids, including theoretical ones describing the constitutive equation for the stress tensor of a magnetic suspension [2,7]. The main feature of these suspensions comes from their ability to vary over a wide viscosity range in a fraction of seconds [8,9]. This feature enables a flow control by an external magnetic field, providing an efficient way to control force or torque transmission [10]. Some recent works [11–17] have also explored not only FHD (ferrohydrodynamics) effects, but also MHD (magnetohydrodynamics) mechanisms for flows over different geometries, using both numerical and theoretical approaches.

Currently, magnetic fluids are used in several technical and medical applications [19]. Ferrofluids are largely used as dynamic seals in hard disks and dampers in loudspeakers [20]. In magnetic separation they accelerate the process of separating oil from water [21]. Magnetic fluids can also accelerate convective cooling and make it possible in low gravity environments [22]. In medicine, biocompatible magnetic fields can transport drugs inside the human body and by controlling an external applied magnetic field, carry these drugs to a specific desired location [23].

Ferrofluid pipe flow in an oscillating magnetic field along the pipe axis was studied theoretically in a wide range of flow rate by [24]. More recently, ferrofluid pipe flow has been examined numerically to investigate the accuracy of an analytic solution for the magnetization equation and assess its validity when used for non-uniform magnetic fields [25]. The case where the applied magnetic fields along and transverse to the duct axis are spatially uniform and varying sinusoidally with time was explored by [26]. In previous works we have performed computer simulations of magnetic fluid laminar pipe flows showing magnetic field induced drag reduction [21,27]. In this work this idea will be extended using a more robust calculation for the case in which the fluid magnetization is coupled with the flow's vorticity. For this end we develop a study on the influence of an applied magnetic field gradient on a laminar pipe flow of a magnetic symmetric fluid. It is shown how the friction of the flow varies in the presence of a magnetic field gradient under the condition in which the fluid magnetization is coupled with the flow's vorticity. An asymptotic solutions have been also developed in order to verify the numerical predictions. The results in this work are important for understanding magnetic drag reduction in laminar pipe flows by applying a gradient of a magnetic field.

2. Governing equations

The model of pipe laminar flow of a magnetic fluid is based on the coupling between classical hydrodynamics and electromagnetism theories in the magnetostatic limit. Fig. 1 shows a sketch of the geometry and some variables of the problem explored here.

Hydrodynamic equations include the continuity and the balance of linear momentum equations. The former is given by:

$$\nabla \cdot \mathbf{u} = 0\,, \tag{1}$$

where $\mathbf{u}$ represents the velocity field. Continuity equation was simplified considering all compressible effects negligible. Balance of linear momentum is expressed by Cauchy's equation [28]:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \nabla\cdot\boldsymbol{\Sigma} + \rho\mathbf{g}, \tag{2}$$

where ρ denotes the fluid density and $\mathbf{g}$ is the gravity acceleration.

The stress tensor $\boldsymbol{\Sigma}$ results from the sum of hydrodynamic Σ_h and magnetic $\boldsymbol{\Sigma}_m$ effects,

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_h + \boldsymbol{\Sigma}_m \tag{3}$$

The stress tensor for the hydrodynamic contribution is given by the constitutive equation for a Newtonian fluid [33]:

$$\boldsymbol{\Sigma}_h = -P\mathbf{I} + 2\eta\mathbf{D} \tag{4}$$

where P represents the hydrodynamic pressure, η is the suspending liquid viscosity, $\mathbf{I}$ is the unit second rank tensor and $\mathbf{D}$ represents the second rank tensor associated with the strain rate of the flow.

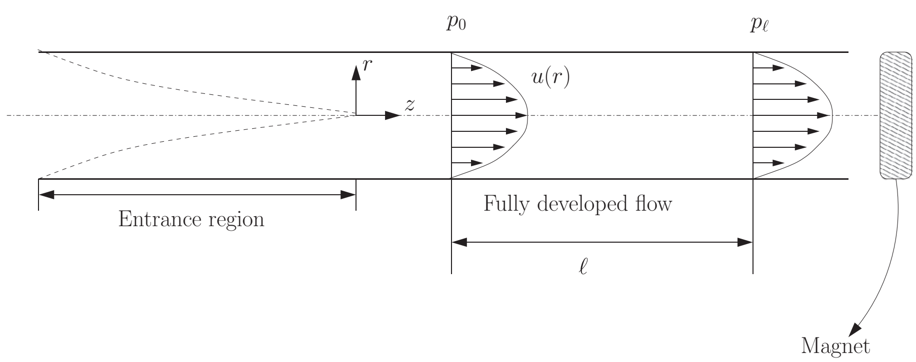

Fig. 1. Sketch of the flow problem geometry studied in this work.