Applied Mathematical Modelling 40 (2016) 4023–4031

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Local bifurcation analysis of a rotating blade☆

Xiaohua Zhang[a,d,*], Fangqi Chen[b], Baoqiang Zhang[c], Taiyan Jing[d]

[a] School of Information Technology, Jiangsu Institute of Commerce, Nanjing 210016, China
[b] Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[c] School of Aerospace Engineering, Xiamen University, Xiamen 361005, China
[d] Department of Mechanics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

ARTICLE INFO

Article history:
Received 18 November 2014
Revised 24 October 2015
Accepted 6 November 2015
Available online 12 November 2015

Keywords:
Blade
Stability
Bifurcation
Center manifold
Normal form

ABSTRACT

In this paper, the stability and local bifurcation for the rotating blade under high-temperature supersonic gas flow are investigated using analytical and numerical methods. Based on obtained four-dimensional averaged equation for the case of 1:1 internal resonance and primary resonance, two types of critical points for the bifurcation response equations are considered. The points are characterized by a double zero and two negative eigenvalues and two pairs of purely imaginary eigenvalues, respectively. For each type,the steady state solutions and the stability region is obtained with the aid of center manifold theory and normal form theory. We find the Hopf bifurcation solution which indicates the blade will flutter. In summary, the numerical solutions, whose initial conditions are chosen in the stability region, agree with the analytic results.

1. Introduction

Rotating blades with spinning axis can operate in the high speed and high temperature gas flow environment [1]. However, the high speed airflow and the high temperature gas flow may affect on the behaviors of the blade obviously. For example, the high speed airflow could cause static and dynamic instabilities of the blade; the high temperatures gas flow may affect the dynamic response characteristics of the blade dramatically [2–4]. Therefore, the research on the dynamic behaviors of rotating blades under high temperature and speed is of great practical importance.

Many investigations related to the dynamic behaviors of rotating blades in high speed and high temperature gas flow environment have been carried out. Considering the effects of transverse shear deformation and rotational inertia, Chen and Chen [5] studied the vibration and stability of thick rotating blades with a single edge crack. In order to reduce the resonant vibration of the blade, Wang and Shieh [6] assumed the friction coefficient of the damper to be a function of the sliding velocity of the contact surface and studied the influence of the friction coefficient. Chen and Peng [7] investigated the dynamic stability of a rotating blade subjected to axial periodic forces with Galerkin finite-element method. Lin and Chen [8] took advantage of finite-element method and Hamilton's principle to study the dynamic behaviors of the pretwisted laminated rotating blade subjected to the periodic axial force. Yao et al. [9] exploited isotropic constitutive law, Hamilton's principle, Galerkin's approach and the method

☆ This work was supported by the National Natural Science Foundation of China (11172125, 11572148) and the National Research Foundation for the Doctoral Program of Higher Education of China (20133218110025).

* Corresponding author. Tel.: +86 25 15996231221.
E-mail address: zhangcy11@163.com (X. Zhang).

http://dx.doi.org/10.1016/j.apm.2015.11.008

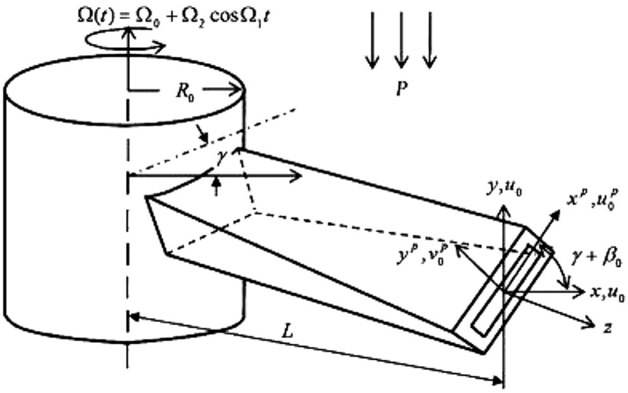

Fig. 1. The model of the rotating blade.

of multiple scales to derive the four-dimensional averaged equation for the case of 1:1 internal resonance and primary resonance and studied the nonlinear dynamic response of the rotating blade using numerical methods.

In addition, much work related to complex dynamic bifurcation phenomena exhibited by a nonlinear autonomous system in the vicinity of compound critical points have been discussed by a few researchers in detail. With the application of the intrinsic harmonic balancing [10] and unification technique [11,12] which leads to various static and dynamic solutions, Yu and Huseyin studied the stability and bifurcation behaviors of the systems with the critical points characterized by a double-zero eigenvalues [11], a simple zero and a pair of pure imaginary eigenvalues [12], two distinct pairs of pure imaginary eigenvalues [13] and so on [14,15]. Later Yu et al. [16–21] used the symbolic language Maple based on the method of multiple time scales for computing the corresponding normal forms. The approach is computationally efficient and systematic. Based on the approach, the stability and local bifurcation of many systems are investigated. Wang et al. [22] considered the stability of a model of a flexible beam undergoing a large linear motion with combination parametric resonance. Zhang et al. [23] investigated the local bifurcation of a functionally graded material plate under transversal and in-plane excitations.

In this paper, we extended Yao, Chen and Zhang's work [9] to investigate the stability and local bifurcation behaviors of the blade under high-temperature supersonic gas flow through both analytical and numerical approaches. All the two types of critical points will be studied in detail. These points are characterized by a double zero and two distinct negative eigenvalues and two pairs of pure imaginary eigenvalues in nonresonant case, respectively. The stability conditions and transition curves are obtained by using center manifold theory, normal form theory, bifurcation and stability theory. We find the Hopf bifurcation solution which indicates the blade will flutter. Flutter can lead to catastrophic structure damage, so the reasonable structure parameters should be designed to avoid it. All numerical results agree with the analytic predictions, which are found by using the symbolic computation language Maple.

This paper is organized as follows: in Section 2, the averaged equations of blade with varying rotating speed are given in terms of a set of differential equations and the stability conditions of initial equilibrium solution are obtained explicitly. The detailed stability and bifurcation analysis of the system in the vicinity of the two types of critical points are presented. In Section 3, which is followed in Section 4 by a short summary.

2. Formulation of the problem

Fig. 1 shows a pretwisted flexible cantilever blade with length L mounted on a rigid rub with radius R_0. The blade is allowed to vibrate flexurally in the plane that makes a setting angle γ. Here, the rotating blade is treated as a pretwist, presetting, thin-walled rotating cantilever beam. β_0 is the pretwist at the beam tip and $\Omega(t)$ is the rotating speed. The origin of the rotating coordinate systems (x, y, z) is locates at the blade root. The local coordinates (x^p, y^p, z^p) are defined, where x^p and y^p are the principal axes of an arbitrary beam cross section. Using the isotropic constitutive law, Hamilton's principle and Galerkin's approach, the nonlinear dimensionless two-degree-of-freedom system is obtained as follows [9],

$$\begin{aligned} &\ddot{p} + \omega_1^2 p + \varepsilon(\beta_{12}\dot{p} + \beta_{13}\dot{q} + \beta_{11}q - 2\beta_{14}p\Omega_0\Omega_2\cos\Omega_1 t - \beta_{14}p\Omega_2^2\cos^2\Omega_1 t + \beta_5 pq^2 + \beta_5 p^3) = \varepsilon\beta_{16}\Omega_1\Omega_2\sin\Omega_1 t \\ &\ddot{q} + \omega_2^2 q + \varepsilon(\beta_{22}\dot{p} + \beta_{23}\dot{q} + \beta_{21}p - 2\beta_{24}q\Omega_0\Omega_2\cos\Omega_1 t - \beta_{24}q\Omega_2^2\cos^2\Omega_1 t + \beta_5 p^2 q + \beta_5 q^3) = 0, \end{aligned} \tag{1}$$

where p and q are two modes derived by using Galerkin method. The modes considered are the typical form of the free vibration for the cantilever beam. Since the problem here is a weak nonlinear case, the mode of the free vibration can be considered as an approximation to the mode of the weak nonlinear system. β_{11}, β_{21} and β_{22} are related to the supersonic gas flow and the geometrical dimensions of the rotating blade. All the other parameters are combined parameters and can be found in [9]. With the method of multiple scales, Yao et al. transformed the two-degree-of-freedom system into the averaged equations in