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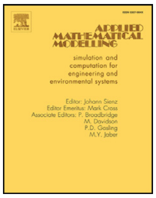

Weighted proper orthogonal decomposition of the swirling flow exiting the hydraulic turbine runner

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ABSTRACT

In this paper we propose a framework for orthogonal decomposition of swirling flows applied to problems originating from turbomachines, where dynamics of the swirling flow in the draft cone is strongly influenced by the turbine discharge coefficient. A weighted proper orthogonal decomposition method (wPOD) is proposed for analyzing the evolution of the swirling flow exiting the hydraulic turbine runner as the turbine discharge is modified. The chief idea is that through orthogonal decomposition one can better identify the leading modes of the axial and circumferential velocity profiles perturbation with respect to a simple base flow. Moreover, it is expected that only one mode is actually responsible for the stability loss. The key innovation introduced in this paper resides in identification of two perturbation quantities having vanishing integrals on the computational domain. By applying the weighted POD on these perturbation quantities, the property of vanishing integral is conserved for each individual mode. As a result, the POD representation of the velocity field is achieved with a number of modes significantly lower compared with other classic techniques. The efficiency of the reduced model developed in this paper is tested by comparing the computed solution with the experimentally measured profiles. In addition, a qualitative analysis of the reduced order model by its correlation coefficient and root mean squared error (RMSE) is performed.

1. Introduction

In the field of modern hydraulic turbines, despite the advent and maturation of high-performance computing, Direct Numerical Simulations (DNS) solvers remain so computationally intensive that they cannot be performed as often as needed (see for reference the work of Buron et al. [1], Guenette et al. [2], Nicolet et al. [3], Ruprecht [4], for turbomachinery problem examples).

Alternative numerical techniques of modeling the fluid-structure interaction in turbomachinery have been proposed, each having its advantages and disadvantages. For example, a methodology underlying the calculation of axisymmetrical turbulent flow in the diffuser cone of the hydraulic Francis turbine was presented by Resiga and his collaborators [5] and the comparison with Laser Doppler Anemometry measurements [6] confirms the correctness of the numerical results. Tridon et al. [7] proposed a novel flow control technique focused on discharge imbalance mitigation in Francis turbine draft-tube bays. Furthermore, accurate numerical simulations are dealing with very high dimensional systems. To overcome these limitations, solution algorithms based on spectral methods have been proposed in our previous investigations [8,9] and a reduced order model based on a

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Nomenclature

r	radial coordinate
r_w	dimensionless radial distance to the cone wall
q	dimensionless discharge coefficient
m	dimensionless flux of moment of momentum
$v_a^{(q)}(r), v_u^{(q)}(r)$	axial and circumferential velocity profiles
$\left(\bar{v}_a^{(q)}, \bar{v}_u^{(q)}\right)$	base flow
$\left(\tilde{v}_a^{(q)}, \tilde{v}_u^{(q)}\right)$	perturbation part
$f^{(q)}(r) \equiv r\tilde{v}_a^{(q)}(r)$	perturbation quantity
$g^{(q)}(r) \equiv r^2\tilde{v}_a^{(q)}(r)\tilde{v}_u^{(q)}(r)$	perturbation quantity
N	number of experimentally measured velocity profiles
n	number of grid points in $r-$ direction
$w_\ell,\ \ell = 1, \ldots, N$	the weights of the snapshot matrix
$\phi_j(r),\ j = 1, \ldots, N$	the POD basis functions
$a_j^{(q)},\ j = 1, \ldots, N$	the POD coefficients
p	number of the retained POD modes
$u_{POD}^{(q)}(r)$	the reduced order solution
$RMSE_u^{(q)}$	the root mean square error between the experimental data u and its reduced order solution mapped onto the computational domain
$C_u^{(q)}$	the correlation coefficient between the experimental data u and its reduced order solution
$\lambda_j, j = 1, \ldots, N$	eigenvalues of POD decomposition
$E_p = \sqrt{\sum_{j=p+1}^{N} \lambda_j^2}$	projection error of order p in terms of Frobenius norm

List of abbreviations

POD	proper orthogonal decomposition
wPOD	weighted proper orthogonal decomposition
DNS	direct numerical simulations
SPIV	stereoscopic particle image velocimetry
ROM	reduced order model
BEP	best efficiency point
EVD	eigenvalue decomposition
CVT	centroidal Voronoi tesselation
EVD-wPOD	weighted POD algorithm based on eigenvalue decomposition
EVD-POD	classic POD algorithm based on eigenvalue decomposition
RBF	Radial basis function
wPOD-RBF model	reduced order model based on weighted POD and RBF interpolation
RMSE	the root mean square error

two-level differential quadrature algorithm was derived in [10] to study interactions between roughness structures and concentrated vortices.

Evaluation of the hydraulic turbine efficiency for the whole range of admissible discharge, is possible in the early stage of design by the standard experimental investigation on a turbine model. The real challenge is to determine the velocity profiles exiting the runner without a priori knowledge of the runner design, aiming to develop further solutions to stabilize the flow in the discharge cone. Two directions for investigating the velocity profiles cohabit in the recent studies. Most papers existing in the literature consider theoretical swirl configurations. For instance, Wang and Rusak [11] provide a new study of the axisymmetric vortex breakdown phenomenon, based on a thorough investigation of the axisymmetric unsteady Euler equations. Their study is limited to inviscid and axisymmetric swirling flows and does not consider the interaction with the vortex breakdown phenomenon. In a previous work, Resiga and his coworkers proposed a suitable analytical representation of the swirling flow as a superposition of three distinct vortices, taking the discharge coefficient as independent variable [12]. In the opposite manner, investigators consider velocity profiles fitted by experimentally measured data. The averaged velocity field inside the inter-blade channel of a propeller turbine runner measured using a stereoscopic particle image velocimetry (SPIV) technique is addressed by Aeschlimann [13] and his coworkers. A particle image velocimetry (PIV) system is used by Iliescu et al. [14] to investigate the flow velocity field in the case of a developing cavitation vortex at the outlet of a Francis turbine runner. Both DNS numerical simulation based on theoretical characterization of swirling flow and performing the experimental measurements in situ are not