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A review on nonlocal elastic models for bending, buckling, vibrations, and wave propagation of nanoscale beams

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ABSTRACT

The enabling emerging technologies such as nanotechnology increased the demand for small-size devices. The proper understanding of the nonclassical behavior of nanostructures is key for the design of these devices. As a result, the static and dynamic behavior of nanoscale beam structures has received a great attention in the past few years. This review aims at directing the light to research work concerned with bending, buckling, vibrations, and wave propagation of nanobeams modeled according to the nonlocal elasticity theory of Eringen. Due to the large body of references found in the literature, the authors chose to briefly present the key findings and challenges and direct light to possible future work. This review does not intersect with recent relevant reviews, which reflects its significance to readers.

1. Introduction to nonlocal continuum mechanics

Nanotechnology and nanoscience helped introduce structures and devices with good precision at nanoscale. Nanobeams are the core structures widely used in many systems such as nanosensors and actuators for sensing and energy harvesting applications. Due to the great potential of nanoscale systems for enhancing many engineering applications, their mechanical behavior should be investigated and well identified before new designs can be proposed. The classical continuum theories prove to predict the response of structures up to a minimum size below which they fail to provide accurate predictions. To account for the small-scale effect, nonlocal continuum theories have been proposed. The nonlocal theories add a size parameter in the modeling of the continuum. This paper is concerned with models developed according to the widely used nonlocal elasticity theory of Eringen [1–6,246].

Eringen developed a unified foundation for the basic field equations of nonlocal continuum theories. According to Eringen's theory, the nonlocal continuum mechanics differs from classical continuum mechanics in two basic aspects:

(i) the proposed global form of the energy law;
(ii) the stress at a point is influenced by the strain at all other points at all past times (memory dependence).

Investigating the significance of nonlocality, Eringen postulated that the classical continuum theories are applicable inside a domain of length and time scales. If L refers to an external characteristic length such as crack length and l denotes an internal

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characteristic length, then classical theories are applicable in the region $L/l \gg 1$. If $L/l \sim 1$, then classical theories fail to predict accurate results and either atomistic or nonlocal models should be used. Similarly in time-dependent problems, the validity of the classical theories is related to the ratio of an external characteristic time T to an internal characteristic time τ.

Early formulations of the nonlocal elastic constitutive equations using lattice dynamics are given by Kröner [7], Kunin [8], and Krumhansl [9]. Continuum approaches to memory-dependent nonlocal elasticity are made by Eringen [10], Eringen and Edelen [11] and Eringen [246] who formulated the nonlocal constitutive equations and included the balance laws in a global form which was not considered by early theories. Gao [12] developed a general theory of nonlocal elasticity based on the nonlocal continuum field theory developed by Eringen [4]. This theory showed that the general model of nonlocal elasticity is asymmetric due to the nonlocal effect of local rotation and anisotropy. According to Gao [12], the higher gradient model can be reduced from the nonlocal theory and the couple stress theory is a special case of higher gradient theory. Chen et al. [13] showed that the nonlocal continuum theories can be physically interpreted from the molecular dynamics perspective.

The nonlocal elasticity is referred to as integral or strongly nonlocal when the stress at a point is influenced by the entire strain field of a material. For weakly nonlocal theories, the stress is expressed as a function of the local strain and its gradient. Eringen's model of nonlocal elasticity proves to be a promising criterion in the analysis of nanostructures taking into account the effect of small size. Wang and Liew [14] showed that small scale effect is not evident for structures with dimensions in the order of micrometers, whereas it is noticeable for nanostructures.

Many authors applied the nonlocal effects to modify classical beam theories in order to analyze static, buckling and dynamics of nanobeams such as Reddy [15,16], Aydogdu [17], Zhang et al. [98], Civalek et al. [28], Roque et al. [18], Thai [19], Thai and Vo [20], Berrabah et al. [21], Ghannadpour et al. [22], Pradhan and Mandal [23], and Challamel et al. [24]. The authors used analytical and numerical solution methodologies such as Ritz analytical method, Fourier series, differential quadrature method (DQM), finite element method (FEM), and finite difference method. Lim and Wang [25], Wang and Liew [14], Wang et al. [26] and Lim and Xu [247] discovered significant over/under-estimation of stress levels using the lower-order nonlocal model, particularly at the vicinity of the clamped end of a cantilevered nanobeam under a tip point load. Using an exact variational principle, they presented an asymptotic model of the nonlocal beam.

Applicability of modeling a linear and nonlinear bending behavior of carbon nanotubes (CNTs) by nonlocal beam theories are discussed by Gao and Lei [27], Arash and Ansari [242], Civalek and Demir [28], Ghasemi et al. [29], Pradhan [30], De Rosa and Franciosi [31], Shen and Zhang [32], Phadikar and Pradhan [33], Pradhan [30], and Mahmoud et al. [34]. Alshorbagy et al. [35] investigated static behavior of nonlocal nanobeams using the FEM, Mahmoud et al. [34] and Juntarasaid et al. [36] studied the static deflection of nanobeams where the effects of the nonlocal elasticity and surface properties are considered. Barretta and De Sciarra [37] studied static behavior of nanobeams based on adopted thermodynamic approach for various beam theories.

The aim of this review is to briefly summarize the state of the art and highlight possible future work of the response of nanobeams modeled according to Eringen nonlocal elasticity theory. The review is concerned with the bending, buckling, vibration, and wave propagation of beams modelled according to the nonlocal elasticity theory of Eringen. Deep investigation into the different approaches of the nonlocal modeling is outside the scope of this work. In a recent paper, Arash and Wang [38] presented a review on the nonlocal elastic models and their applications to graphenes and carbon nanotubes. Out of all references cited in their review, only a few of them are cited in this review which ensures the difference in scope. The paper is laid down so as to present bending of nanobeams in Section 2, buckling in Section 3, vibrations in Section 4, and wave propagation in Section 5. Section 6 presents a unified model for nonlocal elastic beams according to the classical, first-order, and higher-order beam theories. Concluding remarks are finally presented in Section 7.

2. Static analysis

The effect of small size on the bending due to static loading has been thoroughly investigated in the literature [14,16,19,40,42]. Peddieson et al. [41] studied the bending of Euler–Bernoulli cantilever beams and showed the potential application of nonlocal elasticity in nanotechnology. In their analysis, they gave an expression for the ratio of the nonlocal maximum deflection δ to the maximum local deflection δ_0 as

$$\delta/\delta_0 = 1 + (n\, e_0 a/L)^2 \tag{2.1}$$

which shows that the maximum nonlocal deflection depends on the ratio of internal characteristic length $e_0 a$ to the beam length L, where n is an integer.

Reddy and Pang [42] modeled carbon nanotubes according to Eringen's nonlocal theory using Euler–Bernoulli theory (EBT) and Timoshenko beam theory (TBT). They derived the linear bending equation for a nonlocal Euler–Bernoulli beam under a distributed transverse load $q(x)$ in the form

$$EI\,\frac{\partial^4 w}{\partial x^4} + \mu\,\frac{\partial^2 q}{\partial x^2} - q = 0 \tag{2.2}$$

while for a nonlocal Timoshenko beam, it takes the form

$$\frac{\partial}{\partial t}\left[G\,A\,K_s\left(\frac{\partial w}{\partial x} + \varphi\right)\right] - \,\mu\,\frac{\partial^2 q}{\partial x^2} + q = 0 \tag{2.3}$$