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Non-linear fracture analysis of Cantilever Beam Opened Notch specimen

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ABSTRACT

The present theoretical study deals with non-linear fracture behavior of Cantilever Beam Opened Notch (CBON) specimen. It is assumed that the mechanical response of the CBON specimen can be described by the stress–strain diagram of an elastic-perfectly plastic material for both tension and compression. Non-linear fracture analysis is performed applying the *J*-integral approach. For this purpose, a model based on mechanics of materials is used and analytical solution of *J*-integral useful for a different level of stresses and strains in the beam is found. The influence of crack position as well as crack length on *J*-integral value is investigated. The analysis also shows that the inclusion of material non-linearity increases the value of *J*-integral.

1. Introduction

Interlaminar fracture is one of the main mechanisms of failure for laminated materials like unidirectional fiber reinforced composites. Therefore, the important task of Fracture mechanics concerning with laminates is to develop methods for theoretical and experimental investigation of interlaminar cracks. For this purpose, beam or plate configurations are used [1–6].

Recently, the Cantilever Beam Opened Notch (CBON) specimen has been developed [7]. The CBON is used for analysis of interlaminar fracture in unidirectional fiber reinforced polymer composites. The interlaminar crack is situated in the middle of the beam cross-section, parallel to the reinforcing fibers (Fig. 1). The load is a vertical force F applied at the beam free end. It should be specified that the essence of the CBON is the fact that the fixed support constraints the lower crack arm and, thus, the upper crack arm is stress free.

The methods of linear-elastic fracture mechanics are applied and the following formula for strain energy release rate is obtained [7]:

$$G = \frac{21F^2(L-a)^2}{4Eb^2h^3}, \tag{1}$$

where E is the longitudinal modulus of elasticity.

However, it should be noted that the new invented composites possess high delamination fracture toughness [8–10] and the applicability of linear-elastic fracture mechanics is often limited by the fact that the plastic deformation occurs prior to macroscopic delamination crack growth. Therefore, the fracture analysis should take into account the material non-linearity.

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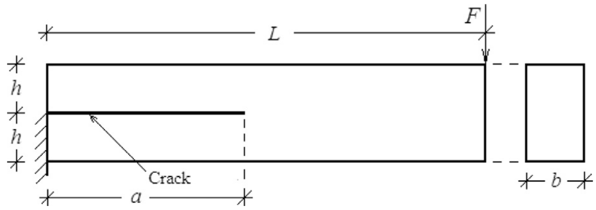

Fig. 1. Cantilever Beam Opened Notch (CBON) specimen.

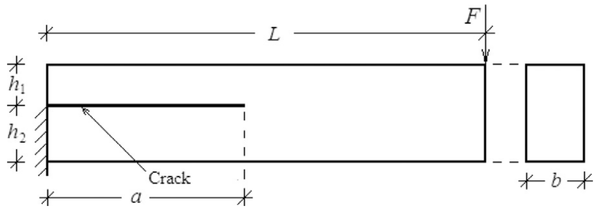

Fig. 2. CBON specimen with different heights of the crack arms.

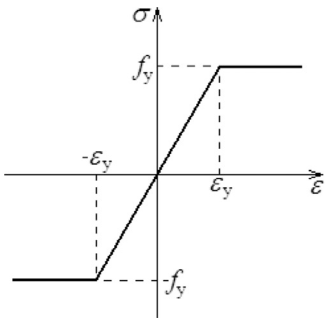

Fig. 3. Idealized stress–strain diagram of an elastic-perfectly plastic material.

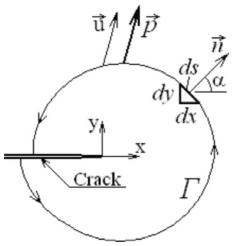

Fig. 4. Elements of *J*-integral.

Hence, the purpose of the present article is to perform non-linear fracture analysis of CBON where the crack arms have different heights (Fig. 2).

In order to perform the investigation, the material building CBON is supposed to follow the stress–strain relation of an elastic-perfectly plastic material (Fig. 3).

Besides, since the deformation theory of plasticity [11,12] is applied in the analysis, it is assumed that the magnitude of the external force increases only. This means that the active deformation of the beam is considered [13,14].

The investigation is performed by means of *J*-integral, a fracture mechanics parameter introduced by Rice [15] and defined as:

$$J = \int_{\Gamma} \left\{ u_0 \cos\alpha - \left[p_x \frac{\partial u}{\partial x} + p_y \frac{\partial v}{\partial x} \right] \right\} ds, \tag{2}$$

where u_0 is the strain energy density, p_x and p_y are the components of the traction vector on the contour Γ, u and v are the displacements along the axes x and y, ds is the infinitesimal part of the contour (Fig. 4). Integration begins from lower crack arm, goes along contour Γ in counterclockwise direction and finishes in upper crack arm. The important feature of *J*-integral is that its value does not depend on the integration path chosen.