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Comparison of two volume balance fully implicit approaches in conjunction with unstructured grids for compositional reservoir simulation

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ABSTRACT

In reservoir simulation, compositional modeling is one of the most commonly used approaches for enhanced oil recovery processes. The methods used to solve the equations arising from the modeling of fluid flow in the reservoirs involve the degree of implicitness and the selection of the primary and secondary equations; primary and secondary variables have a great impact on the computational time. In this work, we implement and compare two fully implicit methods based on volume balance approach. The two methods share the same set of primary variables: pressure and total number of moles of each component. The total number of moles of each component is solved with use its material balance equation, whereas the pressure is solved with use of a volume balance equation. The difference between the two methods is in the nature of the volume balance equation. Whereas for one of the formulations the volume balance equation is the volume constraint and hence the only terms that appear in the Jacobian matrix are those from the volume in which the volume balance is evaluated, the second formulation considers an expanded form of the volume constraint. The main advantage of this expanded equation is that the Jacobian matrix involves information from the volume in which the balance is performed and from all neighboring volumes. The element-based finite-volume method in conjunction with unstructured grids for 2D and 3D reservoirs is used to discretize the material and volume balance equations. For two dimensions, quadrilateral and triangular elements are considered, whereas for three dimensions, hexahedral, prismatic, tetrahedral, and pyramidal elements are considered. The implementations were performed with the UTCOMP simulator developed at the University of Texas at Austin. We compare the performance of the two above-mentioned fully implicit formulations with the implicit pressure explicit composition (IMPEC) formulation of the UTCOMP simulator. The results of several case studies are compared in terms of volumetric oil and gas rates and the total CPU time. The results show good agreement between the production rates and saturation fields for all formulations. Additionally, the performance of the fully implicit methods was superior to that of the IMPEC method as a larger number of grid blocks were used in the simulations.

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1. Introduction

Petroleum reservoir simulation is an important tool used to forecast the oil and gas production rates as well as the amount of fluid that resides in the reservoir. In general, the modeling of the oil recovery processes consists of mass, volume, and energy balances that can reproduce the fluid dynamics inside the reservoir with the desired degree of realism and accuracy. These material balances form a nonlinear differential set of equations that cannot be solved without the use of numerical approaches unless several simplifications are made.

The reservoir simulation has evolved greatly since its introduction, not only in the physical modeling equations used but also in many other features, such as numerical formulations, gridding, flux approximation schemes, phase behavior calculations, geomechanics models, fractures and fault models, and linear solvers. Fussel and Fussel [1] were the first authors to develop a simulator using an equation of state for both phase equilibrium and density calculations. They used the constraint equations (equilibrium equations and volume constraint) to solve n_c+1 primary variables and the flow equations to solve the secondary variables. Coats [2] presented the first fully implicit (FI) formulation for the isothermal compositional model. He used a Gaussian elimination to decouple the primary variables from the secondary variables. Nghiem et al. [3] developed an implicit pressure explicit saturation (IMPES) formulation that differs from the previously mentioned formulations by solving pressure and compositions separately. Young and Stephenson [4] developed a new approach based on the formulation proposed in [1] which is also an implicit pressure explicit composition (IMPEC) approach. The major difference between these two approaches resides in the selection of the primary variables and in the ordering of the equations. Another FI model was proposed by Chien et al. [5]. In this model, the primary equations are obtained from the material balance equations of each component. They proposed a set of primary variables similar to that proposed by Coats [2], except that gas mole fractions were replaced by the equilibrium ratios (K values). Ács et al. [6] proposed a new IMPES formulation that shares the primary variables of Kazemi et al. [7] and Nghiem et al. [3]. Although the pressure equation is based on a volume balance as in the other two models, it is obtained in a special way that allows the formulation to perform just one flash calculation per time step. Watts [8] combined the one iteration per time step idea of Ács et al. [6] with the sequential implicit pressure and saturations (IMPSAT) formulation of Spillette et al. [9] to generate a new IMPSAT formulation. Quandalle and Savary [10] extended the formulation of Watts to solve an inconsistency of this formulation. They included the solution of n_c-2 new variables in the material balance equations. The new variables can be solved in terms of oil or gas compositions. Collins et al. [11] presented an adaptive implicit approach for an isothermal compositional formulation. The equations for this formulation are the n_c+1 material balances and the volume constraint. The primary variables are the total number of moles per bulk volume of the n_c components and water. Branco and Rodriguez [12] proposed a new IMPSAT formulation based on the formulation of Coats [2]. Wang et al. [13] proposed a new FI formulation. In this formulation the flow equations and the equilibrium constraints are all assembled into the Jacobian matrix. Haukas et al. [14] improved the approach of Quandalle and Savary [10] by changing the primary variables. Haukas et al. [15] gave a better interpretation of these parameters. They called the new parameters "isochoric parameters." A stability criterion was also given in [15]. Santos et al. [16, 17] implemented and compared the following approaches: the FI formulations of Coats [2], Collins et al. [11], and Wang et al. [13], the IMPSAT formulation of Branco [12], and an IMPES formulation. Fernandes et al. [18] compared the formulation of Ács et al. [6] with the formulation of Watts [8].

Most of the formulations presented previously used Cartesian grids in conjunction with the finite-volume method (FVM). However, all the formulations can be implemented for any spatial discretization since their derivations are independent of the grid discretization. However, as the Cartesian grid is the simplest way to discretize the domain, the complexity of the implementation of a given formulation for other types of grids will increase sharply. The unstructured grids are more general in terms of modeling important features of the reservoirs. The unstructured grids are usually related to the concept of elements. However, for many years this concept was used only in the finite-element method (FEM) until the pioneer work of Baliga and Patankar [19] that combined the conservative approach of the FVM with the idea of elements and shape functions of the FEM, creating a new method that they named the "control volume finite-element method" (CVFEM). Later, Maliska [20] suggested that the CVFEM denomination is unsuitable, since the CVFEM gives the wrong idea that we have a finite-element approach that is based on material balance. Maliska [20] suggested that "element-based finite-volume method" (EbFVM) is a more appropriate denomination since we still have an approach that locally respects the material balance of the physical property being transported. For this reason, in the rest of this article, we will always refer to this approach as the EbFVM.

The first use of unstructured grids in reservoir simulation was by Heinemann and Brand [21] and Heinemann et al. [22] in conjunction with perpendicular bisector (PEBI) grids. These grids are also called "Voronoi grids." Like the Cartesian grids, the PEBI are cell-center grids and therefore are used for isotropic media, and it is possible to evaluate fluxes with use of only two grid points. The first use of the EbFVM in reservoir simulation was by Rozon [23]; he used it to solve a single-phase flow using quadrilateral elements. Rozon [23] also presented a comparison of the truncation errors between the EbFVM and the Cartesian grids, showing that for regular grids composed of quadrilateral elements the EbFVM is more accurate. Fung et al. [24] used PEBI grids based on triangular elements in a thermal general-purpose simulator. Cordazzo [25] solved the two-phase flow (water and oil) in conjunction with the EbFVM using triangular and quadrilateral elements. Marcondes and Sepehrnoori [26] used the EbFVM for the FI isothermal compositional simulation in conjunction with triangular and quadrilateral elements. Recently, Marcondes et al. [27] and Santos et al. [28] implemented the EbFVM for 3D isothermal compositional reservoir simulation using four element types: hexahedron, tetrahedron, pyramid, and prism. Also, using the EbFVM approach, Fernandes et al. [29] has investigated the use of several interpolation functions in conjunction with 2D compositional reservoir simulation. More recently, Fernandes et al.