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Duality system-based derivation of the modified scaled boundary finite element method in the time domain and its application to anisotropic soil

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ABSTRACT

In this study, an efficient method is proposed for the dynamic analysis of a two-dimensional semi-infinite soil with rigid bedrock, which is applicable to the cross-isotropic and anisotropic soil models. The original scaling center is replaced by a scaling line, so the modified scaled boundary finite element method (SBFEM) is more suitable for analyzing the horizontal layered soil. For the first time, the dual system is employed to derive the displacement equation for the modified SBFEM. By introducing the dual variables, the governing equations are derived in the framework of a Hamilton system. Next, the dynamic stiffness equation is obtained according to the weighted residuals method. The displacement equation of motion for the far field is built by applying the continued fraction method and introducing auxiliary variables. Based on the sub-structure method, the far field can be seamlessly coupled with the near field. Importantly, the efficient and precise time-integration method is first employed to solve the global equation of motion. High computational precision can be achieved using the proposed method. An extremely efficient and accurate solution can be obtained by applying this method to solve the equation of motion for the modified SBFEM. Finally, the accuracy and high efficiency of the proposed method is demonstrated for the anisotropic soil model based on numerical examples.

1. Introduction

In practical engineering, most soils exhibit anisotropic material characteristics, as described in previous studies, e.g., Ward et al. [1], Pickering [2], and Arthur and Menzies [3]. Studying the dynamic responses of anisotropic soils is of fundamental importance in several branches of engineering, such as nuclear power plant design, earthquake engineering, and the prevention and control of traffic vibration. Thus, it is necessary to consider the anisotropy of soil materials from the perspective of engineering applications. However, it should be noted that most previous studies have assumed that soils are isotropic and linearly elastic. Several reviews have discussed the problems of soil structure interactions in an isotropic medium, such as Luco [4], Gazetas [5], and Wolf [6]. In recent decades, significant advances have been made in applying the dynamic

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analysis of soil structure interaction problems to isotropic soil, but there is still a need to establish the method for treating anisotropic soil problems. In most cases, due to the original deposition, microstructural characteristics, and accumulated overburden pressure, the material axes of the soil might not coincide with the Cartesian space coordinate axes; therefore, the soil exhibits the mechanical property of anisotropy. Wolf [7] is generally considered to be the first to have investigated the anisotropic characteristics of soil, but many studies have been performed since soil anisotropy theory was proposed [8–12]. However, these studies did not focus on dynamic analysis based on anisotropic soil as a foundation because the governing equations for elastic wave propagation in anisotropic soil are complex, and there are also difficulties obtaining their accurate analytical solutions. To overcome these difficulties, the generalized anisotropic nature of soils can be idealized by a reduced form of anisotropy, which is known as cross-anisotropy, where its properties are symmetric about the vertical axis [13,14]. The dynamic analysis of cross-anisotropy in soil overlaying rigid bedrock has been investigated by many civil engineers. However, to the best of our knowledge, no previous studies have analyzed the dynamic response of anisotropic soil overlaying rigid bedrock, which is an interesting subject for future research.

In recent years, many numerical approaches have been proposed for the dynamic analysis of an unbounded domain, as discussed in several reviews [15–19]. These numerical approaches can be summarized as follows. The finite element method [20,21] is an effective method for the numerical analysis of an unbounded domain, which employs viscous-spring artificial boundaries to model a propagating wave that generally decays slowly. By applying the sub-structure method, the overall problem domain can be divided into the far field and near field with small finite elements. However, the near field must be sufficiently large to obtain accurate results. Therefore, the amount of computer storage and the data preparation time required are usually large, especially for large scale systems. Recently, the boundary element method (BEM) [22,23] emerged as an effective numerical method for solving a wide range of engineering problems, which is used widely in many fields, such as nonlinear problems and stress singularity problems. As its name suggest, only the boundary of the problem domain is dispersed for the linear elastic problem, and thus the problem dimensionality is reduced by one. However, this method has been applied only rarely to dynamic analysis involving anisotropic soil, mainly because the BEM is not effective for solving the appropriate Green's functions for the anisotropic elastodynamics problem. The thin layer method (TLM) [24–26] is only suitable for horizontal layered soil, where the analytical solution is obtained in the direction of the horizontal layer by constructing the exact non-reflecting boundary condition. As mentioned above, the TLM has been successfully used for modeling the simple geometry of a homogeneous material.

The scaled boundary finite element method (SBFEM) [27–29] is a semi-analytical technique, which combines the advantages of the BEM and finite element method, such as the lack of requirement for a fundamental solution and artificial boundary conditions. Only the boundary is discretized, so the spatial dimension is reduced by one. The displacement equations in the SBFEM are formulated according to the principle of virtual work or by using the weighted residual method. By introducing the same shape function, the SBFEM can be seamlessly coupled with the finite element method in a straightforward manner. Furthermore, it can be used to model the dynamic analysis of anisotropic media without any difficulties.

The SBFEM has been applied successfully to the dynamic analysis of soil–structure interactions in the frequency domain [30,31] and time domain [32–34]. By applying the inverse Fourier transform, the dynamic stiffness equation of the SBFEM is transformed into the unit-impulse response matrix equation in time domain, where time discretization is required to solve the integral equation. To obtain a correct solution, the time history must be discretized into quite small steps, which requires a large amount of computational time. In addition, the computational efficiency tends to decrease as the scale of the problem increases. Therefore, many researchers have aimed to improve the computational efficiency of this method [35–38]. The continued fraction method is a novel technique for solving the dynamic problem in the time domain, which has attracted much attention. Song [39] first proposed the continued fraction method for a bounded domain. In order to overcome the unstable low frequency component, Birk et al. [40,41] developed a high-order doubly asymptotic boundary for the acoustic and diffusion problems. Subsequently, Chen [42,43] and Birk [44] solved the bounded and unbounded domain problems by applying the continued fraction method in the time domain.

In this study, we propose a method for obtaining a continued fraction solution for a semi-unbounded soil with bedrock in the time domain. Based on the Hamiltonian variational principle, the SBFEM displacement equation, which is a linear second order differential equation, can be obtained in the frequency domain. Next, the first order differential equation of dynamic stiffness is derived by applying the weighted residuals method. Finally, the dynamic stiffness is expressed by the continued fraction method. The standard equation of motion for structural dynamics is formulated by introducing high order static stiffness, mass matrices, and expansion terms. Many numerical integration methods can be used to solve the equation of motion, such as the Newmark method and Wilson-θ method, which are step-by-step integration methods [45,46]. Zhong proposed the precise integration method (PIM) in order to overcome the disadvantages of these methods, such as exponent overflow and instability [47–49]. The integration step is divided into quite small pieces and each small interval is approximated by five Taylor expansion terms. Therefore, higher computational precision can be achieved using this method, which greatly avoids rounding off errors. The equation of motion for the modified SBFEM can be solved correctly and efficiently by applying the PIM.

The remainder of this paper is organized as follows. The modified SBFEM dynamic stiffness equation is derived in the framework of a duality system in Section 2. The technique used in the derivation process is first applied to the modified SBFEM. The continued fraction method for the modified SBFEM is described in Section 3. The dynamic stiffness is expressed as a series of continued fractions. The coupling of the near field and far field in the time domain analysis using the