Applied Mathematical Modelling 40 (2016) 5324–5334

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analysis of the dynamics of thin isotropic cylindrical shell in asymptotic approach

A. Bhattacharyya*, B. Mukhopadhyay

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India

ARTICLE INFO

Article history:
Received 5 June 2014
Revised 25 July 2015
Accepted 30 September 2015
Available online 30 December 2015

Keywords:
Linear classical theory
Asymptotic approach
Isotropic body
Cylindrical shell
Cylindrical symmetry
Radial and longitudinal vibration

ABSTRACT

In this paper, a dynamic behavior of an isotropic cylindrical shell under cylindrical symmetry is presented by asymptotic approach. Here some special assumptions are set to make the problem simple. An attempt is taken to give an analytic expression of radial vibration of a semi-infinite cylinder. In this problem it is assumed that the thickness of the shell is so small that variants of the vibrations exhibit infinite power series expansion across the thickness. As a result of this assumption it is shown that all modes of variants remain uncoupled and satisfy the same equations of motion approximately.

1. Introduction

Theory of vibrations and waves in elastic medium was well established by Love [1]. Later Leissa [2] published a collection of works on vibrations of shell. Mirsky [3] presented the propagation of wave in transversely isotropic circular cylinder of infinite length. Gazis [4] studied the waves in a hollow cylinder of infinite length in the most general form. Study on axisymmetric wave propagation in circular cylindrical shell, immersed in fluid was done by Sinha et al. [5]. Wang et al. [6] discussed the vibration of functionally graded multi-layered orthotropic cylindrical panel under thermo-mechanical load. Sharma [7] examined the three dimensional vibration of a homogeneous transversely isotropic thermo-elastic cylindrical panel. Analysis of free vibration of transversely isotropic piezoelectric circular panels was performed by Ding et al. [8]. Soldatos and Hadhgeorgian [9] worked on the frequency of vibration in isotropic cylindrical shell and panel by iterative approach. Free vibration of composite cylindrical panels with random material properties was advanced by Sing et al. [10] where they modeled the mechanical properties of laminated composite cylindrical panel on its natural frequencies as random variables. Zhang [11] explained a wave propagation method to analyze the frequency of cylindrical panels. Loy and Lam [12] analyzed the vibrations of thin cylindrical panels with simply supported boundary conditions by Flugge's theory and also discussed the vibration of rotating cylindrical panel. Kapuria et al. [13] obtained three dimensional solution of a simply supported rectangular hybrid plate. Discussion on natural frequency of a cylindrical panel on a Kerr foundation was presented by Cai et al. [14]. Free vibrations of thin cylindrical shells having finite lengths with freely supported and clamped edges were analyzed by Yu and Syracuse [15]. The static and dynamic analysis of plates supported on elastic foundations [16] has become an interesting problem in Engineering. The dynamic response of isotropic cylindrical shell buried

* Corresponding author. Tel.: +91 3325658505.
E-mail address: aritramathnet09@gmail.com (A. Bhattacharyya).

http://dx.doi.org/10.1016/j.apm.2015.09.103
S0307-904X(15)00646-0/

at a depth below the free surface of the ground has been shown by Wong et al. [17] by applying three-dimensional elastic theory. By employing membrane theory, Paliwal et al. [18] displayed with clarity the coupled free vibrations of isotropic circular cylindrical shell on Winkler and Pasternak foundations. Upadhyay and Mishra investigated the non-axisymmetric dynamic behavior of buried orthotropic cylindrical shells [19] excited by a combination of P-, SV- and SH-waves.

Pioneering contribution of Kirchoff brought about a clear understanding of the foundations of classical plate theory. After that the theory was really developed and many publications were published where the foundations and methods of deduction of Kirchoff–Love theory and its possible improvements were discussed. The theory of small deflection of thin elastic plate is based on the assumptions which are presented in the book of Timoshenko and Woinowsky-Krieger [20]. One of the successful methods of deduction is the two-dimensional approximation of three dimensional theories of elastic plate. It has been deduced by considering priori assumptions regarding the variations of unknowns (i.e. displacements and the stresses) across the thickness of the plates. In the method of asymptotic expansion, a formal power series expansion of three-dimensional solution is used by considering the thickness of the plate as a small parameter. Few references and books related to this can be found in [21].

In this paper we are going to analyze free vibration of an isotropic thin cylindrical shell under cylindrical symmetry by asymptotic approach. [22–25] can be mentioned as examples of recent works in this direction. Here we formulate our problem in a simple way to reduce the complexity of the system of equations which normally occurs in most of the previous works. In this way we can go for a direct analytical solution without making any presumption on the form of solution and then we make an attempt to analyze it.

2. Basic equations and notations

For a homogeneous isotropic elastic medium, balance of linear momentum yields the following equations:

$$t^{ij}{}_{,i} + \rho(f^j - \ddot{u}^j) = 0, \tag{1}$$

and balance of moment of momentum yields,

$$t^{ij} = t^{ji}, \tag{2}$$

where $i, j = 1, 2, 3$; ρ is the density; t^{ij}, f^i, u^i are the contra-variant component of stress-tensor, body-force density, displacement of the continuum along x_i-direction respectively. Here 'dot' denotes the differentiation with respect to time t and 'comma' denotes the covariant differentiation with respect to x_i-co-ordinate.

The following constitutive equations are given by:

$$t^{ij} = \lambda \varepsilon^k_k g^{ij} + G(\varepsilon^{ij} + \varepsilon^{ji}), \tag{3}$$

where λ, G are the two elastic constant in classical theory for isotropic body and $\varepsilon^i_j = u^i{}_{,j}$, g^{ij} is fundamental metric-tensor; and $i, j, k = 1, 2, 3$.

Here we use cylindrical co-ordinate and let $\widehat{r}, \widehat{\theta}, \widehat{z}$ be the unit vector forming ortho-normal triad at the point (r, θ, z). $t_{rr}, t_{\theta\theta}, t_{zz}, t_{rz}, t_{\theta z}, t_{r\theta}$ are the components of stress tensor and u_r, u_θ, u_z are the components of displacement vector with respect to unit ortho-normal triad mentioned above. They are given by,

$$\begin{aligned} &g_{11}t^{11} = t_{rr},\ g_{22}t^{22} = t_{\theta\theta},\ g_{33}t^{33} = t_{zz},\ \sqrt{g_{11}g_{22}}t^{12} = t_{r\theta},\ \sqrt{g_{33}g_{22}}t^{32} = t_{z\theta},\ \sqrt{g_{11}g_{33}}t^{13} = t_{rz}, \\ &\sqrt{g_{11}}u^1 = u_r,\ \sqrt{g_{22}}u^2 = u_r,\ \sqrt{g_{33}}u^3 = u_z. \end{aligned} \tag{4}$$

For cylindrical co-ordinate,

$$x^1 = r,\ x^2 = \theta,\ x^3 = z,\ g_{11} = 1,\ g_{22} = r^2,\ g_{33} = 1 \text{ and } \begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = -r,\ \begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \frac{1}{r},$$

are the only non-zero Christoffel symbols of second kind in cylindrical co-ordinate system.

3. Mathematical formulation

We consider here a thin right circular cylindrical shell and use cylindrical co-ordinate system under which a point in space is represented by (r, θ, z).

Let us suppose that the shell be sufficiently long with one end lying at $z = 0$ plane. It occupies the region between $r = a$ and $r = b$, where $b > a$ and $b - a$ is small. We take z-axis along the axes of the cylinder and want to discuss its vibration under axial symmetry.

We introduce a new variable R defined by,

$$R = \frac{2r - (a + b)}{b - a}.$$

Now as r runs over a to b, R runs over -1 to 1.