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Three-dimensional analytical solutions for the axisymmetric bending of functionally graded annular plates

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ABSTRACT

Based on the three-dimensional theory, the axisymmetric bending behavior of functionally graded annular plates made of magneto-electro-elastic, piezoelectric and elastic materials respectively, was investigated. The transverse loads were expanded in terms of a Fourier–Bessel series and the solutions corresponding to each item of the series were found by the direct displacement method. Then the superposition principle was utilized to determine the mechanical, electric and magnetic quantities of the annular plates subjected to the arbitrary transverse pressures. It was found that four unusual boundary conditions can be rigorously satisfied at the inner and outer circumferential edges by this method. Meanwhile, the common boundary conditions such as simply supported, clamped and free boundary conditions were approximately satisfied using the Saint-Venant's principle. An innovative isoparametric element was also developed to calculate the axisymmetric behavior of the functionally graded magneto-electro-elasticity for comparison. Numerical examples were finally given to validate the present method.

1. Introduction

Circular and annular plates were popular in many engineering fields and they were classic subjects in the theory of elasticity due to its relatively simple geometry. The analytical solutions based on three-dimensional theory were explored for circular and annular plates, but still have limitations so far. The first attempt could be traced back to Celep [1,2], who developed a method to analyze the free vibrations of isotropic circular plates by the three-dimensional theory. After a decade, this method was extended to transversely isotropic and laminated circular and annular plates by Fang and Ye [3–5]. Unfortunately, Ding and co-authors [6–9] found a fatal error involved in this method and proposed the correct one. Based on three-dimensional theory, the piezoelectricity and magneto-electro-elasticity were also exactly solved for the free vibrations and static bending of circular and annular plates. However, these exact solutions were available for some unusual boundary conditions only and not for the common boundary conditions such as simply supported, clamped and free ones. For this purpose, Nie and Zhong [10,11] introduced the semi-analytical method, which was initially developed by Chen and Lü [12] for rectangle plates, to investigate the functionally graded circular and annular plates. This semi-analytical method was extended to investigate the functionally graded solid and annular plates subjected to thermo-mechanical load with various boundary conditions [13]. An exact solution in closed-form was given for the circular/annular functionally graded plates

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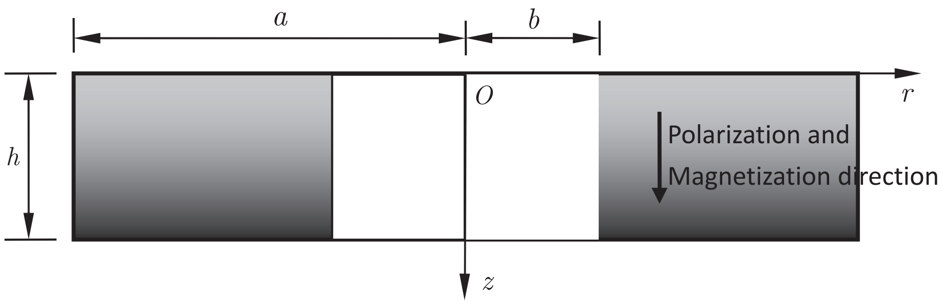

Fig. 1. FG Magneto-electro-elastic annular plate and coordinates system.

embedded in piezoelectric layer based on Reddy's plate theory [14]. Su et al. [15] also presented a three-dimensional analysis for functionally graded conical, cylindrical shells and annular plates using modified Fourier series. Alternatively, Li et al. [16] found three-dimensional analytical solutions for the axisymmetric bending of the functionally graded circular plates subjected to polynomial loads of even order using the stress function method, and extended to magneto-electro-elastic annular plates [17]. Recently, Wang et al. [18] presented the analytical solutions for the axisymmetric bending of functionally graded circular plates subjected to arbitrary transverse loads, which were applicable to both unusual and common boundary conditions. This method was also extended for the piezoelectric and magneto-electro-elastic circular plates [19,20].

On the basis of our previous works [18–20], the three-dimensional analytical solutions were proposed for the axisymmetric bending of the functionally graded magneto-electro-elastic annular plates subjected to arbitrary transverse loads, which were expanded in terms of the Fourier–Bessel series including the second kind Bessel functions. The solution corresponding to the each item of the series was found and superposed for the final solutions according to the principle of superposition. The solutions were also simplified for the piezoelectric and elastic annular plates when the coupling magneto-electro-elastic and coupling piezoelectric effect were ignored. Finally an innovative element was developed for the axisymmetric behavior of the functionally graded magneto-electro-elasticity plate. The numerical examples were illustrated and compared with those available in literatures.

2. Magneto-electro-elastic annular plates

2.1. Statement of the problem

A transversely isotropic magneto-electro-elastic annular plate is considered as shown in Fig. 1. The inner radius is b. The outer radius is a. The thickness is h, and the symmetrical axis of the material is parallel to the z-axis as well as the polarization and magnetization direction. The origin of the coordinates (r, θ, z) is located at the center of the top surface of the plate. The transverse loads were acted symmetrically on the top and bottom surfaces of the plate. The governing equations of the problem can be referred to as Eqs. (1)–(14) in Ref. [20], and are not repeated herein for brevity.

2.2. The Fourier–Bessel expansion of the transverse loads

A function $H_\mu(\cdot)$ is defined as

$$H_\mu(k\xi) = AJ_\mu(k\xi) + BY_\mu(k\xi) \quad (\mu = 0, 1), \tag{1}$$

where $J_\mu(\cdot)$ and $Y_\mu(\cdot)$ are the first and second kind Bessel functions of μth order. A and B are two parameters related to the boundary conditions at the circumferential edges. The axisymmetric load acting on the plates is denoted by $P(\xi)$ with ξ in the range $0 < \xi_0 \le \xi \le 1$. If the function $P(\xi)$ satisfies the Dirichlet's condition, it can be expanded in the following series [21]

$$P(\xi) = \sum_i C_i H_0(k_i\xi), \tag{2}$$

in which the Fourier coefficient C_i is

$$C_i = \bar{C} \int_{\xi_0}^{1} \xi P(\xi) H_0(k_i\xi) d\xi, \tag{3}$$

where the parameter $\bar{C}_i$ is determined by the parameters k_i, A and B, and the detail can be found in the Appendix A.

If the solutions corresponding to $C_iH_0(k_i\xi)$ are found, the solution to the arbitrary load $P(\zeta)$ can be obtain using the principle of superposition. As a result, the load is assumed in the form $C_iH_0(k_i\xi)$ and the parameter k_i is represented by k in the following derivation for simplicity.