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Modeling and analysis of a non-autonomous single-species model with impulsive and random perturbations

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ABSTRACT

In this paper, we study the dynamic behaviors of a non-autonomous single-species model subject to impulsive and random perturbations. We first investigate the existence and uniqueness of the globally positive solution of the model. Secondly, a good understanding of the extinction, non-persistence in the mean, weak persistence, persistence in the mean and stochastic permanence of the model is obtained. Thirdly, we further examine the asymptotic pathwise estimation. Fourthly, it is shown that the model is globally attractive under some appropriate conditions. Finally, numerical simulations are presented to justify the analytical results.

1. Introduction

It is well known that many ecological systems are affected by abrupt changes, and such changes are often assumed to be in the form of impulses in the modeling process. Many systems with impulsive perturbations are the impulsive differential equations, which are extensively used as models in ecology, physics, biotechnology, chemistry and other applied sciences (see [1–9]). Recently, Tan et al. [9] have considered the effect of impulsive perturbations and discussed the periodicity and stability of the following non-autonomous system:

$$\begin{cases} \dfrac{dx(t)}{dt} = x(t)[a(t) - b(t)x(t)] - \dfrac{c(t)x(t)}{d(t)+x(t)}, \ t \neq \tau_k, \\ x(\tau_k^+) = (1+h_k)x(\tau_k), \ t = \tau_k, \ k \in \mathbb{N}, \end{cases} \tag{1.1}$$

where $x(t)$ denotes the population size, the positive coefficients $a(t)$ and $b(t)$ are the intrinsic growth rate and self-inhibition rate, respectively. The term $c(t)x(t)/(d(t)+x(t))$ represents predation, and moreover, it is an increasing function with respect to x and has a saturation value for large enough x. The positive coefficients $c(t)$ and $d(t)$ are measures of the saturation value. The coefficients $a(t)$, $b(t)$, $c(t)$, $d(t)$ are positive, continuous and bounded functions on $\mathbb{R}_+ = [0, +\infty)$. $\mathbb{N}$ represents the set of positive integers. The impulsive points satisfy $0 < \tau_1 < \tau_2 < \cdots$ and $\lim_{k\to+\infty} \tau_k = +\infty$. In view of biological

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meanings, some natural restrictions are $h_k > -1$, and moreover, when $h_k > 0$, the impulsive effects represent stocking, while $h_k < 0$ denote harvesting.

In fact, the above motivation comes from the models proposed by Ludwig et al. [10], Murray [11] and Liu et al. [12]. In [10], Ludwig et al. introduced a single-species system with the form:

$$x'(t) = x(t)[a - bx(t)] - \mathscr{H}(x). \tag{1.2}$$

Here the biological meanings of $x(t)$, a and b are the same as that of system (1.1). The term $\mathscr{H}(x)$ is predation which is an increasing function and saturates for large enough x. To be specific, Murray [11] took the form for $\mathscr{H}(x)$, that is, $cx^2(t)/(d + x^2(t))$, and discussed the asymptotic behavior of the following system,

$$x'(t) = x(t)[a - bx(t)] - \frac{cx^2(t)}{d + x^2(t)}, \tag{1.3}$$

where c and d are measures of the saturation value.

Considering the effect of periodic variation environment and impulsive perturbation on system (1.3), Liu et al.[12] investigated the periodicity and stability of system:

$$\begin{cases} \dfrac{dx(t)}{dt} = x(t)[a(t) - b(t)x(t)] - \dfrac{cx^2(t)}{d + x^2(t)}, & t \neq \tau_k, \\ x(\tau_k^+) = (1 + h_k)x(\tau_k), & t = \tau_k, \ k \in \mathbb{N}. \end{cases} \tag{1.4}$$

To further investigate the effects of other specific forms of $\mathscr{H}(x)$, similar to Murray [11], Tan et al. [9] took $c(t)x(t)/(d(t) + x(t))$ and studied the periodicity and stability of the above system (1.1) by using the method in Ref. [12].

Note that the population dynamics in the real world is inevitably affected by environmental noise. A system with such random perturbations tends to be suitably modelled by stochastic differential equation. Just as Nisbet and Gurney [13] have pointed out that stochastic differential equation models play a significant role in various dynamical system analysis, as they can provide some additional degree of realism compared to their deterministic counterpart. In recent years, the dynamical behaviors of many stochastic systems have attracted great attention and have been studied extensively, see some books [13–15] and papers [16–22].

Inspired by the above ideas, we assume that the white noise affects the intrinsic growth rate mainly with

$$a(t) \to a(t) + \sigma(t)\dot{W}(t), \tag{1.5}$$

where $\dot{W}(t)$ is a white noise, namely, $W(t)$ denotes the standard Brownian motions defined on this probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with the usual conditions. $\sigma(t)$ is a bounded and nonnegative function on $t \geq 0$ and $\sigma^2(t)$ represents the intensity of the white noise. Then it follows from (1.1) and (1.5) that a stochastic system can be described by the form:

$$\begin{cases} dx(t) = x(t)\left[a(t) - b(t)x(t) - \dfrac{c(t)}{d(t) + x(t)}\right]dt + \sigma(t)x(t)dW(t), & t \neq \tau_k, \\ x(\tau_k^+) = (1 + h_k)x(\tau_k), & t = \tau_k, \ k \in \mathbb{N}, \end{cases} \tag{1.6}$$

where initial value $x(0) > 0$.

In this paper, we will focus on the asymptotic behaviors of system (1.6). Our motivation comes from the work of Liu and Wang [16] in which they discussed system (1.6) without predation (that is, $c(t)x(t)/(d(t) + x(t)) \equiv 0$). In this contribution, we investigate the influences of predation, impulsive and stochastic perturbations on the asymptotic behaviors of system (1.6), and a good understanding of extinction, non-persistence in the mean, weak persistence, persistence in the mean, stochastic permanence, pathewise estimation and global attractivity is obtained. To the best of our knowledge, there are few published papers concerning system (1.6). The rest structure of this paper is as follows. In Section 2, some preliminaries are introduced. In Section 3, we study the existence and uniqueness of positive solution of system (1.6). Sufficient conditions for the extinction, non-persistence in the mean, weak persistence, persistence in the mean, stochastic permanence are established in Section 4. Pathwise estimation and global attractivity are discussed in Sections 5 and 6, respectively. In Section 7, we present some specific numerical examples to verify the theoretical results. Conclusions are given in Section 8 and we interpret how impulsive and stochastic perturbations lead to significant changes in the population size of species. We give some proofs of Theorems in appendices for convenience in reading this paper.

2. Preliminaries

In this section, we shall state some definitions and lemmas which will be useful for establishing our main results. For convenience and simplicity in the following discussion, we always use the notations:

$$g_* = \inf g(t), \quad g^* = \sup g(t), \quad t \in \mathbb{R}_+,$$

where $g(t)$ is a continuous bounded function. Throughout this paper, we assume that there exist a pair of positive constants h and H such that for all $t > 0$:

$$h \leq \prod_{0<\tau_k<t} (1 + h_k) \leq H. \tag{2.1}$$