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Lorentz covariance of heat conduction laws and a Lorentz-covariant heat conduction model

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ABSTRACT

Lorentz covariance is one of the two basic assumptions in relativity and it has considerable universality in nature. Lorentz covariance has been investigated widely in various fields including thermodynamics, but studies of heat conduction remain very limited. In this study, we demonstrate that several typical heat conduction laws, i.e. Fourier's law, the Cattaneo–Vernotte (CV) model, and Jeffery model, are not Lorentz-covariant. Thus, we propose a new heat conduction model that satisfies Lorentz covariance. Compared with the existing heat conduction laws, which are not Lorentz-covariant, this new Lorentz-covariant model can ensure that singularity and violation of the second law of thermodynamics do not apply in any inertial reference system. The CV model and the new model both predict thermal wave phenomena, but the CV model predicts a pure wave whereas the new model predicts a composite of translational motion and a wave. Therefore, the new Lorentz-covariant model makes the features of heat conduction more coherent.

1. Introduction

Lorentz covariance is one of the two basic assumptions of special relativity and it is also considered to have considerable universality in nature. In fluid dynamics and other fields involving macroscopic statistics, Lorentz covariance has been investigated widely, especially with respect to the basic equations in these fields [1–4]. In equilibrium thermodynamics, fairly in-depth investigations have been performed for the Lorentz transformations of temperature, entropy, pressure, and other thermodynamic quantities [5–10]. In non-equilibrium thermodynamics, the classical Fourier's law of heat conduction is often used to describe classical heat conduction problems. However, the limitations of Fourier's law have been elucidated in recent years [11–19]. One of the major problems is that Fourier's law predicts an infinite speed of heat perturbation propagation, which apparently violates relativity [20]. In addition, Fourier's law of heat conduction cannot predict the supertransient and high heat flux processes well [11–15].

Several modified heat conduction models have been proposed to overcome these limitations. The Cattaneo–Vernotte (CV) model [21–22] is the main example, which predicts the hyperbolic heat conduction equation and wave-like transport in heat conduction processes called thermal waves. The CV model agrees well with many experiments and simulation data [12–13]. The Jeffrey model [12] can be considered as an extension of the CV model, which includes the influence of temperature relaxation. Tzou [23] proposed a single-phase-lagging heat conduction model, which can reduce to the CV model by making a first-order Taylor series approximation. Anisinov et al. [24] proposed a model for metals based on the interactions between

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electrons and phonons, and Guyer et al. [25] provided a model for pure phonon heat conduction. There have also been further modifications and improvements to these classical models. For example, Tzou [26] proposed a dual-phase-lagging model to include the influence of temperature lag in his single-phase-lagging model, while Coleman et al. [27] improved the changing rate of the heat energy in the CV model. Most of these models are linear and they predict limited speed in heat conduction processes, which can overcome the infinite speed problem in Fourier's law. In addition, some nonlinear models can address this problem, such as the thermomass theory [28–32] for heat conduction based on relativity and the mass-energy equation, as well as alternative approaches to the analysis of the diffusion equation [33–35]. These models are nonlinear, but they are closer to Fourier's law and more concise than some linear models. In particular, in alternative approaches to the analysis of the diffusion equation, the equation can be changed into Burger's equation [33–35], which means that some existing conclusions in mathematics can be used to analyze heat conduction problems.

These modified heat conduction models can overcome the infinite heat propagation speed problem in Fourier's law, which violates relativity, but this does not mean that they obey relativity. A limited speed of heat conduction does not equal the Lorentz covariance required by relativity. In essence, a general heat conduction process can be viewed as the result of electromagnetic interactions and the classical electromagnetic theory is Lorentz-covariant [36]. In addition, the thermomass theory of heat conduction is based on relativity and the mass-energy equation, and thus it also requires that the heat conduction is Lorentz-covariant. Therefore, it is necessary that heat conduction laws obey Lorentz covariance. Unfortunately, investigations of this issue have been very limited.

In this study, we first discuss the Lorentz covariance of typical existing heat conduction models and we show that they are not Lorentz-covariant. Next, we propose a new heat conduction law that satisfies Lorentz covariance. We also provide detailed discussions of some aspects of this new model, including its prerequisites and the Lorentz transformation of physical properties, as well as its advantages, physical meaning, and comparisons with other heat conduction models. Based on these discussions, we show that heat conduction laws should be Lorentz-covariant to ensure that singularity, such as unlimited thermal conductivity, and violation of the laws of thermodynamics, such as negative relaxation time or negative thermal conductivity, do not apply in any inertial reference system. Therefore, the Lorentz-covariant heat conduction model can make the theoretical system of heat conduction more coherent. Finally, we discuss the traveling wave solutions of the CV model and the new model, where we show that although they both predict thermal wave phenomena; the CV model predicts a pure wave whereas the new model predicts a composite of translational motion and a wave.

2. Lorentz covariance of typical heat conduction laws

2.1. Fourier's law of heat conduction

There are four main viewpoints regarding the Lorentz transformation of temperature and heat [5–10,37–41]: (1) Plank–Einstein's transformation $T' = \sqrt{1-\beta^2}T$ [6], (2) Ott's transformation $T' = \frac{T}{\sqrt{1-\beta^2}}$ [9], (3) Landberg's view $T' = T$ [10], and (4) Newburgh's view [37] that there is no universal Lorentz transformation of temperature. Plank–Einstein's transformation can be deduced by the thermodynamic method [38] and statistical mechanics method [39]. However, Ott [9] proposed another opposite Lorentz transformation of temperature in 1963, and many arguments and different views have appeared subsequently. Landberg [10] stated that temperature is Lorentz invariant, while Newburgh [37] suggested that there is no universal Lorentz transformation of temperature and that each Lorentz transformation has its own specific condition. In the present study, we do not discuss this problem with respect to the first three views as the difference between the transformations is only the constant coefficient of temperature, which has little influence on the results of the present study. As a case study, we consider Ott's Lorentz transformation of temperature $T' = \frac{T}{\sqrt{1-\beta^2}}$ and heat $Q' = \frac{Q}{\sqrt{1-\beta^2}}$. The Lorentz transformations of differential operators are $\frac{\partial}{\partial x'} = \gamma(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t})$, $\frac{\partial}{\partial t'} = \gamma(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x})$. Q is the quantity of heat, $q = \frac{\partial Q}{\partial t}$ is the heat flow, and T is the temperature in a certain inertial reference system. Q', T', and q' are the corresponding physical quantities in another inertial reference system, respectively. u is the relative velocity between the two inertial reference systems, with $\beta = \frac{u}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. We can obtain

$$q' = \frac{\partial Q'}{\partial t'} = \gamma\left(\frac{\partial Q'}{\partial t} + u\frac{\partial Q'}{\partial x}\right) = q + u\frac{\partial Q}{\partial x}, \tag{1}$$

$$\frac{\partial T'}{\partial x'} = \frac{\partial T}{\partial x} + \frac{u}{c^2}\frac{\partial T}{\partial t}. \tag{2}$$

For one space dimensional problems, Fourier's heat conduction law is $q = -\lambda\frac{\partial T}{\partial x}$. Its Lorentz transformation should have the same form $q' = -\lambda'\frac{\partial T'}{\partial x'}$. Substituting Eqs. (1) and (2) into this formula yields:

$$q + u\frac{\partial Q}{\partial x} = -\lambda'\left(\frac{\partial T}{\partial x} + \frac{u}{c^2}\frac{\partial T}{\partial t}\right). \tag{3}$$

We note that Fourier's law only contains the heat flow and temperature gradient, but Eq. (3) contains the heat flow and temperature gradient as well as the rate of temperature change and the energy field gradient. Thus, it is difficult to guarantee