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An efficient plate element for the vibration of composite plates

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ABSTRACT

An element is presented for the free vibration analysis of composite plates. The element is capable of representing high orders of displacement using very few degrees of freedom. Hierarchy in the orders of deflection, rotation, moment and shear has been preserved in the element through scrupulous choice of field variables. The effect of rotary inertia has been considered through an innovative scheme for inclusion into mass lumping procedures. Performance of the element has been tested in a wide range of problems which indicate that free vibration analysis of plates with the help of the present element can reach a high level of accuracy at reduced computation cost, compared to standard elements in literature.

1. Introduction

Laminated composite plates are being increasingly used in the construction, automotive, marine and aerodynamic industry due to the high stiffness and strength-to-weight ratios, long fatigue life, resistance to corrosion, and the flexibility in design that these materials offer. A true understanding of their dynamic behaviour primarily requires the natural frequencies to be determined accurately. Finite element method is considered the most versatile and efficient tool for analysis of laminated composite structures. While classical laminated plate theories are too simple and inadequate for the analysis of composites, higher order or 3D continuum theories often have a complicated formulation and are computationally expensive. The elements based on Reissner–Mindlin's theory provides a balance between these, and is the most preferred solution to composite plate bending problems, when through-thickness stress and strains are not deemed to be so important.

There has been a wide literature on the theoretical and applicative aspects of composite plates. A survey by Zhang and Yang [1] reviews the recent development of finite elements for the analysis of composite plates. Yang et al. [2] and Reddy [3] were among the first to devise analytical and finite element solutions for the analysis of composite plates with special emphasis on the effect of shear deformation and rotary inertia. Kim and Gupta [4] used a quadratic interpolation function with five degrees of freedom per node to analyse the effects of lamination, shear, and coupling of extension-bending and twist-curvature on the lowest frequencies and corresponding mode shapes. Neogi et al. [5] used a nine-noded Lagrangian

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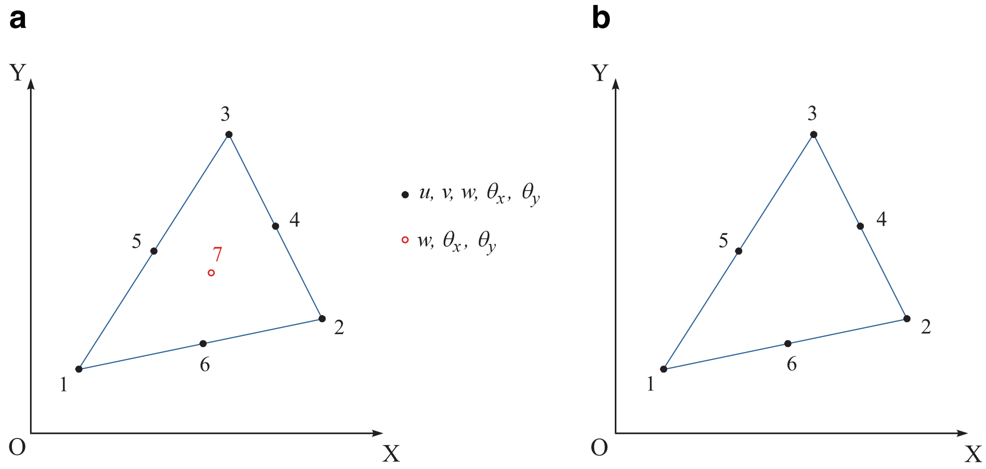

Fig. 1. Nodes and degrees of freedom of the element in the (a) initial stage (b) final stage (after condensation).

plate bending finite element with five degrees of freedom per node and included the effect of rotary inertia to analyse the free and forced vibration response of laminated composite folded plate structures. A mixed finite element for plates and shells was proposed by Gendy et al. [6] to study the effect of inertia operators for vibration of laminated composite plates and shells. Shiau and Chang [7] have presented a higher order plate element for the of free vibration analysis of angle ply rectangular laminates. The total displacement was expressed as the sum of the displacements due to bending and that due to shear. The separate stiffness and mass matrices representing the two displacements were then concatenated and systematically reduced to consider the total deflection. Ghazi et al. [8] have used the Lagranges equations of motion coupled with the finite element technique to analyse free vibration of polygonal composite plates. An eighteen degree of freedom triangular element was modified to develop a higher order thirty six degree of freedom element by considering transverse shear effects. Sivakumar and Iyengar [9] analysed the free vibration of laminated composite plates with cut-outs undergoing large deflections. They have used the Ritz finite element model over a nine noded isoparametric quadrilateral element with higher order displacements to account for parabolic variation of transverse shear stresses. Krishna Reddy and Palaninathan [10] have studied free vibration of skew laminated plates by deriving the consistent mass matrix in explicit form. They have used a high precision triangular plate bending element, where the effect of skew angle, fibre orientation, number of layers and stacking sequences on the fundamental frequency have been studied. Ferreira et al. [11] has used the first-order shear deformation theory in the multiquadric radial basis function procedure for predicting the free vibration behaviour of moderately thick symmetrically laminated composite plates. The transverse deflection and two rotations of the laminate are independently approximated with the help of radial basis functions which is applied to solve partial differential equations using a mesh-free method. Aydoğdu and Timarci [12] has used the Ritz method with polynomial displacements to analyse antisymmetric angle-ply laminates with different boundary conditions. The effect of various parameters such as fibre orientation, number of layers, and boundary conditions on the natural frequencies has been studied, taking two opposite edges free. Carrera and Brischetto [13] has been investigated the effect of thickness locking for a different plate theories, namely thin plate theory, first order shear deformation theory, higher order theories, mixed theories and layer-wise theories. Bending and vibration problems have been analysed for isotropic, orthotropic and multi-layered, composite plates leading to some important conclusions about the different theories. Cong et al. [14] presents a similar radial basis function collocation technique for the free vibration analysis of laminated composite plates using the first order shear deformation theory. Shojaee et al. [15] have developed an isogeometric finite element method based on non-uniform rational B-splines basis functions for free vibration and buckling analysis of thin symmetrically laminated composite plates. Grover et al. [16] has extended the hyperbolic shear deformation theory to analyze free vibration of laminated composite and sandwich plates. Euler–Lagrange equations are derived employing the principle of virtual work for the dynamic problem. A C^0-continuous isoparametric biquadratic-quadrilateral serendipity element is used for the finite element solution of generalized higher order shear deformation theory. Chalak et al. [17] has investigated bending and free vibration behaviour of laminated soft core skew sandwich plates using a C^0 finite element model based on higher order zigzag theory. Carrera [18] proposed a Unified Formulation to generate governing equations and finite element matrices that remain invariant even when the plate theory changes. Using the unified formulation Carrera and Petrolo [19] obtained the closed form solution for isotropic and laminated plates. A 'best plate theory' diagram was developed for several problems including laminated plates, in which the different plate theories were brought on an equal footing in terms of error on stress, displacement etc. By tracing such a diagram, the authors concluded that a number of near-optimal solutions close to the best theory is possible, notwithstanding the fact that each of these individual formulations can differ from each other substantially.

The element presented in this paper has an arbitrary triangular shape with seven nodes in the initial stage — three at the vertices, three at the mid-sides, and a seventh node at the centroid of the element (see Fig. 1 (a)). Each of the six external