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On the phase- lag Green–Naghdi thermoelasticity theories

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ABSTRACT

In this work, we proposed three models of generalized thermoelasticity: a single –phase -lag Green–Naghdi theory of type III, a dual-phase–lag Green–Naghdi theory of type II and of type III. The solid is assumed to be linear anisotropic inhomogeneous. A unified heat conduction law and heat transport equation are given, which consolidate the three theories and also the Lord–Shulman theory and the Green–Naghdi theory of type II. Dissipative inequality is derived, constitutive equations and thermodynamic restrictions are given. The speed of thermal wave propagation is given, for each of the three theories in case of isotropic solid. A uniqueness theorem is proved for the considered theories. Variational characterization of solution is established. An application is given for isotropic thermoelastic solid and the results are presented graphically.

1. Introduction

The classical thermoelasticity theory is that with no coupling between mechanical and thermal fields. In addition, the heat transport equation is of parabolic type and therefore, predicts infinite speed for the transport of heat in the sense that each thermal disturbance to a body instantly propagates, albeit with distortion and attenuation, to every part of the body, even those remote from the disturbance [1]. So, Fourier's law leads to parabolic differential equation for heat transport, thus implying the paradox of heat conduction: Parts of an initially given local heat pulse will propagate with infinite speed through the body. For practical purposes, the paradox of heat conduction is of no concern, if those parts of the heat pulse that have infinite speed are strongly damped. This is the case in all materials at room temperature. However, at low temperature damping may become unimportant and the predictions of the parabolic heat transport equation become measurable false [2].

Thermal energy can be transmitted by free electrons, by lattice vibrations (phonons) and by electron–phonon and phonon–electron interactions. In the second case, a wave-like energy transport can happen at low temperature and the heat signals propagate as thermal waves. This phenomena is called the second sound and occurs, if phonons travel through crystals, interact with each other and conserve the momentum [2,3]. The term "second Sound" is due to the propagation of sound waves in phonon gas causing thermal waves and also to the similarity in in the sound wave propagation process in gases [3]

Narayanamutri and Dynes observed the second sound in Bismuth at law temperature [4]. Because of the experimental evidences in the support of finiteness of the heat propagation speed "Ackerman et al. [5]" the generalized thermoelasticity

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Nomenclature

a_1, a_2, a_3	Key numbers, each equals to 0 or 1
A_{ij}	$= T_0^{-1} k_{ij}^*$
C_{ij}	$= T_0^{-1} k_{ij}$
C_E	Specific heat at constant strains
C_{ijkl}	Elasticity tensor
c	$= 1/\tau_q$
D_{ij}	$= (a_1 C_{ij} + a_2 \tau_\alpha A_{ij})$
f_i	$= \sigma_{ij} n_j$, surface traction
$\hat{f}_i$	Prescribed surface traction
g_i	$= \theta_{,i} = \dot{\beta}_i$, temperature gradient
h_i	$= \ell * T_0^{-1} q_i$ Entropy flux vector
h	$= h_i n_i$
$\hat{h}$	$= T_0^{-1} \ell * \hat{q}$
k	Thermal conductivity
$k*$	Conductivity rate
k_{ij}	Thermal conductivity tensor
k_{ij}^*	Conductivity rate tensor
n_i	The outer unit vector normal to the surface
Q	Intensity of applied heat source per unit volume
q_i	Heat flux vector
q	$= q_i n_i$
$\hat{q}$	Prescribed heat flux on the boundary
S	Entropy per unit volume
T	Absolute temperature
T_0	Reference temperature
t	Time
U	Internal energy per unit volume
u_i	Components of displacement vector
$\hat{u}_i$	Prescribed displacement on the boundary
v_T	Speed of thermal wave propagation
W_i	$= D_{ij} \beta_j + a_3 A_{ij} \gamma_j$
$\boldsymbol{x}$	$= (x_1, x_2, x_3)$ Position
X_i	Mass force

Greek Symbols

α	Thermal displacement $\dot{\alpha} = \theta$
$\hat{\alpha}$	Prescribed thermal displacement
α_T	Coefficient of linear thermal expansion
β	$= \rho C_E / T_0$
β_i	$= \alpha_{,i} = \ell * \theta_{,i}$
γ	$(3\lambda + 2\mu)\alpha_T$
γ_i	$= \ell * \beta_i$
γ_{ij}	Thermo–Elastic Coupling moduli
δ_{ij}	Kronecker's delta
ε_{ij}	Components of strain tensor
$\in$	Internal energy per unit mass
θ	$= (T - T_0)$, temperature deviation, $\lvert\theta\rvert / T_0 \ll 1$
κ	Thermal diffusivity
$\kappa*$	Diffusivity rate $= \frac{k^*}{\rho C_E}$
λ, μ	Lame' constants
ξ	Internal rate of production of entropy per unit volume
ρ	Mass Density
σ_{ij}	Components of stress tensor
τ	Relaxation time
τ_α	Phase-lag of the thermal displacement gradient