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Aleatory and epistemic uncertainties analysis based on non-probabilistic reliability and its kriging solution

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ABSTRACT

The uncertainties affecting the capability of the structural system are generally classified into aleatory uncertainty and epistemic uncertainty. In this study, aleatory uncertainty is modeled by probability theory, and epistemic uncertainty is modeled by evidence theory. A new reliability analysis model is developed to analyze both uncertainties. According to evidence theory, the uncertain input space is first partitioned into different focal elements which contain the random variables with their joint probability density function (PDF) and the interval variables with the joint basic probability assignment (BPA). Then, according to the non-probabilistic theory based on interval analysis, the bounds of the failure probability for each focal element can be obtained through a successive construction of the performance function of the focal element in two levels and the corresponding reliability analysis. Furthermore, the belief and plausibility measures are obtained by the random reliability analysis. In order to improve computational efficiency, the kriging method is employed to build the surrogate model for the constructed performance function and then based on this surrogate model to implement the estimation of the failure probability. The features of the proposed approach are demonstrated with two practical examples.

1. Introduction

Beyond deterministic analysis, nondeterministic analysis has been increasingly researched and adopted in many structural designs to improve the reliability, robustness and safety against the uncertainty of products [1–4]. Uncertainty can be viewed as the difference between the present state of knowledge and the complete knowledge [5]. Probability theory is an important tool to handle uncertainties, and is widely used in engineering applications. However, for a complicated structural system, we cannot collect enough information for a probabilistic simulation to assess its reliability. Due to our lack of knowledge, the mathematical model may not simulate proper behavior of the real physical system, and system failure can occur in unexpected ways [6].

The scientific and engineering communities usually classify uncertainties into aleatory and epistemic types [7–11]. Aleatory uncertainty, also called objective or stochastic uncertainty, arises from the inherent randomness in the properties or behavior of a physical system or environment. Epistemic uncertainty or subjective uncertainty derives from a lack of knowledge or information about a physical system or environment [12–14]. The aleatory uncertainty quantification problem is usually handled with probability theory that has been popular in many quantification problem schemes. Probability

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based research and applications include robust design [15–17], reliability-based design [18–20], and multidisciplinary optimization under uncertainty [21–24]. Epistemic uncertainty is usually modeled by non-probabilistic theory. Recently, several alternatives for the representation of epistemic uncertainty have been conducted, such as interval analysis [25,26], possibility theory [27,28] and evidence theory [29–31]. Since the 1990s, Ben-Haim [32,33] and Elishakoff [34,35] have introduced the non-probabilistic convex set model and established the corresponding reliability analysis approach. According to this line of thought and interval arithmetic, Guo et al. [36,37] proposed a new non-probabilistic reliability index for the uncertainty represented by interval information.

However, in many engineering applications, both aleatory and epistemic uncertainties are present simultaneously in a structural system. Although evidence theory can deal with both types of uncertainties in its framework, it requires much more computational efforts than the probability theory [38]. Helton et al. [13,39–42] have done a significant amount of research works on both types of uncertainties and employed probability as the mathematical structure to represent them. Du [43,44] established the unified uncertainty analysis (UUA) for a mixture of aleatory and epistemic uncertainties and employed the first order reliability method (FORM) to analyze this UUA. According to the non-probabilistic theory based on interval analysis, Guo [45] established the hybrid probabilistic and interval model (HPIM) to deal with the mixture of random and interval variables. But this model cannot be directly used to analysis the mixture of aleatory uncertainty and epistemic uncertainty represented by evidence theory (i.e. multiple interval variables with their BPAs). And there are not effective solutions for this model. Under the UUA framework, a new approach for a mixture of aleatory and epistemic uncertainties is developed following the thought of the HPIM. Furthermore, an algorithm based on kriging method is introduced to alleviate the computational burden.

The remainder of the paper is organized as follows. In Section 2, the analytical framework of aleatory and epistemic uncertainties is briefly reviewed. In Section 3, firstly, the non-probabilistic reliability theory based on interval analysis is reviewed (Section 3.1). Then, a new unified uncertainty analysis approach is developed to analyze the two types of uncertainties (Section 3.2). In Section 4, the kriging-based algorithm is introduced to efficiently estimate the proposed approach. In Section 5, the two examples are employed to demonstrate the feasibility of the proposed approach and the high-efficiency of the kriging-based algorithm. Section 6 offers conclusions.

2. Representation of both aleatory and epistemic uncertainties

2.1. Evidence theory

Evidence theory, also called Dempster–Shafer (DS) theory, was presented by Shafer [29] on the Dempsters work [30,31]. It can directly handle insufficient information and incomplete knowledge situations. Comprehensive works with detailed presentations of DS theory are in Refs. [29–31].

Let Y denote an epistemic variable and $\mathcal{Y}$ denote the set of all possible values for Y. $\mathcal{Y}$ is also called the sample space of Y. Let $\mathcal{U}$ be the subsets of the set of $\mathcal{Y}$ and $\mathbb{Y}$ be a countable collection of subsets $\mathcal{U}$, which is also the set of focal element for Y.

In evidence theory, the basic probability assignment (BPA) m characterizes the basic propagation of uncertain information and is a function defined for subsets $\mathcal{U}$ of $\mathcal{Y}$. It must satisfy the following three axioms [41]:

$$\begin{cases} m(\mathcal{U}) \geq 0 \text{ if } \mathcal{U} \in \mathbb{Y} \\ m(\mathcal{U}) = 0 \text{ if } \mathcal{U} \notin \mathbb{Y} \\ \sum\limits_{\mathcal{U}\in\mathbb{Y}} m(\mathcal{U}) = 1 \end{cases}. \tag{1}$$

According to the intrinsic flexibility of BBA structure, various formations can be constructed. Three formations of BPA structure are shown in Fig. 1, which are general, Bayesian and consonant BPA structures, respectively. We assume that the BPA structures of the epistemic uncertain variables are only from one source as the Bayesian BPA structure in this study. If the information comes from multiple sources, the so-called rule of combination [46] can be used to aggregate the multiple BPA structures.

When the multiple uncertain variables $Y_1, Y_2, \ldots, Y_{nY}$ have associated evidence spaces $(\mathcal{Y}_1, \mathbb{Y}_1, m_1)$, $(\mathcal{Y}_2, \mathbb{Y}_2, m_2)$, $\ldots (\mathcal{Y}_{nY}, \mathbb{Y}_{nY}, m_{nY})$, then the vector $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_{nY})^{\mathrm{T}}$ also has an associated evidence space $(\boldsymbol{\mathcal{Y}}, \mathbb{Y}, m_Y)$, where (i) $\boldsymbol{\mathcal{Y}} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \cdots \times \mathcal{Y}_{nY}$, (ii) $\boldsymbol{\mathcal{U}} = \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \times \mathcal{U}_{nY}$, $U \in \mathbb{Y}$ with $\mathcal{U}_k \in \mathbb{Y}_k (k = 1, 2, \cdots, nY)$, (iii) $m_{\boldsymbol{Y}}(\boldsymbol{\mathcal{U}})$ is called joint BPA and can be calculated as:

$$m_{\boldsymbol{Y}}(\boldsymbol{\mathcal{U}}) = \begin{cases} \prod\limits_{k=1}^{nY} m_k(\mathcal{U}_k) & \text{if } \boldsymbol{\mathcal{U}} = \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \mathcal{U}_{nY} \in \mathbb{Y} \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where the set $\boldsymbol{\mathcal{U}}$ is called focal set or focal element.

In order to more clearly illustrate the join BPA, two independent epistemic variables Y_1 and Y_2 are taken as an example. Their BPA structures are shown in Fig. 2.

Their joint BPA can be calculated by Eq. (2), and the results are listed in Table 1 and also drawn in Fig. 3.