Applied Mathematical Modelling 39 (2015) 7093–7112

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Three-dimensional half-space problem within the framework of two-temperature thermo-viscoelasticity with three-phase-lag effects

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ARTICLE INFO

Article history:
Received 4 January 2014
Received in revised form 29 January 2015
Accepted 24 February 2015
Available online 14 March 2015

Keywords:
Three-phase-lag model
Thermo-viscoelasticity
Two-temperature parameter
Normal mode analysis
Three-dimensional modeling
Thermal source

ABSTRACT

Present paper deals with the problem of thermo-viscoelastic interactions in a homogeneous, isotropic three-dimensional medium whose surface suffers a time dependent thermal shock. The problem is treated on the basis of three-phase-lag model with two temperatures. Medium is assumed to be unstrained and unstressed initially and has uniform temperature. Normal mode analysis technique is employed onto the non-dimensional field equations to derive the exact expressions for displacement component, temperature fields, stress and strain. The problem is illustrated by computing the numerical values of the field variables for a copper material. Finally, all the physical fields are represented graphically to estimate and highlight the effects of the different parameters considered in this problem.

1. Introduction

The conventional dynamic theory of thermoelasticity rests upon the hypothesis of the Fourier's law of heat conduction in which the temperature distribution is governed by a parabolic-type partial differential equation. As a consequence, the theory predicts that a thermal signal is felt instantaneously everywhere in a body. This implies an infinite speed of propagation of the thermal signals and is unrealistic from the physical point of view, especially for short-time responses. Also, it is now well known that heat transmission at low temperature propagates by means of waves. These aspects have aroused much interest and activity in the field of heat propagation and gave rise to the subject "generalized thermoelasticity." The generalized thermoelasticity theories involve hyperbolic-type governing equations and admit finite speed of thermal signals. Among generalized models, the extended thermoelasticity theory involving one thermal relaxation time proposed by Lord and Shulman [1] and the temperature-rate-dependent theory of thermoelasticity including two relaxation times proposed by Green and Lindsay [2] are familiar to many researchers and many works have been done under these theories.

Providing sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems, Green and Naghdi [3–5] introduced three models, which are subsequently referred to as models I, II and III. GN models include a term called 'thermal displacement gradient' among the independent constitutive variables. When the three theories are linearized, the heat transport equation of GN-I is the same as the classical heat equation, whereas both GN-II and GN-III models admit propagation of thermal signals at finite speed. In model II, the internal rate of production of entropy is

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http://dx.doi.org/10.1016/j.apm.2015.02.045

Symbols

σ_{ij}	components of the stress tensor,
u_i	components of the displacement vector,
ρ	density of the medium,
$\lambda^* = \lambda_e(1 + \alpha_0 \frac{\partial}{\partial t})$,	
$\mu^* = \mu_e(1 + \alpha_1 \frac{\partial}{\partial t})$,	
$\beta_1^* = \beta_{1e}(1 + \beta_1 \frac{\partial}{\partial t})$,	
$\beta_{1e} = (3\lambda_e + 2\mu_e)\alpha_t$,	
$\beta_1 = (3\lambda_e\alpha_0 + 2\mu_e\alpha_1)\frac{\alpha_t}{\beta_{1e}}$,	
$\theta = T - T_0$	thermodynamic temperature,
$\phi = \phi - T_0$	conductive temperature,
$K^* = \frac{c_E(\lambda_e+2\mu_e)}{4}$	material constant characteristic of the theory,
λ_e, μ_e	Lame's constants,
$\alpha_0,\ \alpha_1$	viscoelastic relaxation times,
α_t	coefficient of linear thermal expansion,
a	two-temperature parameter,
T	absolute temperature,
T_0	temperature of the medium in its natural state assumed to be $\lvert\theta/T_0\rvert << 1$,
e_{ij}	components of the strain tensor,
e	cubical dilatation,
c_E	specific heat at constant strain,
k	thermal conductivity,
τ_T	phase lag for the temperature gradient,
τ_q	phase lag for the heat flux,
τ_ν	phase lag for the thermal displacement gradient,
δ_{ij}	Kronecker delta function.

taken to be identically zero, implying no dissipation of thermal energy. This model admits undamped thermoelastic waves in an elastic material and is known as the theory of thermoelasticity without energy dissipation. Model III includes the previous two models as special cases. In this model introducing the temperature gradient and thermal displacement gradient as the constitutive variables, the proposed heat conduction law is of the form $\vec{q}(P,t) = -[k\vec{\nabla}T(P,t) + K^*\vec{\nabla}\nu(P,t)]$ where $\dot{\nu} = T$ and $\vec{\nabla}\nu$ is the thermal displacement gradient. The two positive constants k and K^* are the thermal conductivity and the conductivity rate respectively. K^* (of physical dimension conductivity/time) is a material constant characteristic of the theory.

The next generalization to thermoelasticity is reported by Chandrasekharaiah [6]. This model is specially based on the dual-phase-lag heat conduction law proposed by Tzou [7]. Tzou [7] considered microstructural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed due to phonon-electron interactions on the macroscopic level. A macroscopic lagging (or delayed) response between the temperature gradient and the heat flux vector seems to be a possible outcome due to such progressive interactions. Tzou [7] introduced two-phase lags to both the heat flux vector and the temperature gradient and considered a constitutive equation to describe the lagging behavior in the heat conduction in solids. Here the classical Fourier's law is replaced by an approximation to a modification of the law with two different translations for the heat flux vector and the temperature gradient.

Roychoudhuri [8] has recently established a generalized mathematical model of a coupled thermoelasticity theory that includes three-phase lags for the heat flux vector, the temperature gradient and the thermal displacement gradient. The more general model established reduces to the previous models as special cases. The generalized constitutive equation for heat conduction describing the lagging behavior in this model is $\vec{q}(P,t+\tau_q) = -[k\vec{\nabla}T(P,t+\tau_T) + K^*\vec{\nabla}\nu(P,t+\tau_\nu)]$. Here τ_ν, the delay time in the thermal displacement gradient is also introduced in addition to τ_q and τ_T. Three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering etc., where the delay time τ_q captures the thermal wave behavior (a small scale response in time), the phase-lag τ_T captures the effect of phonon-electron interactions (a microscopic response in space), the other delay time τ_ν is effective since in the three-phase-lag model, the thermal displacement gradient is considered as a constitutive variable. The stability of the three-phase heat conduction equation and the relations among the three material parameters are discussed by Quintanilla and Racke [9].

Effect of internal friction on the propagation of plane waves in an elastic medium may be attributed to the fact that dissipation accompanies vibrations in solid media due to the conversion of elastic energy to heat energy. Several mathematical models have been used by authors to accommodate the energy dissipation in vibrating solids where it is observed that internal friction produces attenuation and dispersion; hence, the effect of the viscoelastic nature of material medium in the