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Variational bounds in composites with nonuniform interfacial thermal resistance

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ABSTRACT

The problem of the effective thermal conductivity for a matrix–fiber composite exhibiting a periodic microstructure is studied. The composite is macroscopically anisotropic with nonuniform interfacial thermal resistance between the phases. First, the asymptotic homogenization method, which was employed recently to address related problems, is applied to recover the expression for the effective thermal conductivity obtained from classical micromechanical approaches. Then, improved bounds for the effective conductivity are obtained from a generalization of the two-dimensional realization of results by Lipton and Vernescu (1996). The bounds depend on the concentration and the conductivity of each phase, as well as on the fibers cross-section geometry and type of periodic distribution, and on the nonuniform interfacial resistance. Results are compared numerically with those obtained via asymptotic homogenization and finite element approaches, showing good agreement in a variety of situations.

1. Introduction

Different homogenization techniques have been developed to investigate the effective conductivity tensor for anisotropic two-phase conductive composites with an interfacial thermal resistance between the phases. A common way of estimating the effective thermal conductivity for heterogeneous materials with known microstructures is to make rigorous full-field numerical simulations. In [1], a finite-element-based multiscale methodology was developed to estimate the effective thermal conductivity of unidirectional fibrous composite materials with an interfacial thermal resistance between the continuous and dispersed components. The results obtained in [1] encouraged the extension of the computational procedure to three-dimensional composite geometries. For example, particulate media composed of monodisperse solid spherical particles distributed in a continuous phase were considered in [2], ordered composite materials reinforced with longitudinally aligned circular-cylindrical anisotropic short fibers were addressed in [3], and the limiting cases of ordered arrays of perfectly-aligned prolate ellipsoids of revolution and circular cylinders, which are thermally

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equivalent, respectively, to the simple cubic array of spheres and the square array of unidirectional fibers, were dealt with in [4].

Furthermore, extensive analytical studies on the effective thermal conductivity of two-phase composite materials with interfacial resistance using different approaches have been performed over the years. It has been shown that the effective thermal conductivity of the heterogeneous material is strongly affected by its composition and structure. In [5], modifications to the original theories of Rayleigh and Maxwell allowed the derivation of expressions for the effective thermal conductivity of composites with a thermal barrier resistance between the continuous matrix phase and dilute concentrations of inclusions with spherical, cylindrical and flat plate geometries, showing its dependence on the volume fraction and size of the inclusion phase. However, five very different macroscopic models developed in [6] showed that the effective thermal conductivity tensor also depends strongly on the relative value of the barrier resistance with respect to the resistance of the components. Variational principles and bounds introduced in [7] describing the effective conductivity tensor for two-phase heat conducting composites with interfacial resistance between phases established once again the importance of basic structural models in theoretical analyses. In [8], accurate results for the effective thermal conductivity of a composite material consisting of periodic arrays of spheres with interfacial resistance were obtained by means of very accurate lattice sums. Another interesting methodology was proposed in [9], where analytical expressions for the transverse thermal conductivities of unidirectional fiber composites with thermal barrier were derived based on the electrical analogy technique and on the cylindrical filament–square packing array unit cell model. The self-consistent field theory was extended in [10] to derive the effective thermal conductivity of mixtures produced by randomly distributing composite spheres throughout a continuum, and having imperfect thermal contact between the surfaces of the spheres and the continuous phase. In [11], an asymptotic approach to simulate the imperfect interfacial bonding in composite materials was proposed by introducing a flexible bonding layer between the matrix and inclusions.

In all the works mentioned previously, the thermal resistance was taken to be uniformly distributed on the interfaces. However, as remarked in [12], the importance of considering nonuniform interfacial thermal resistance is argued. In a recent report [13], the effective shear behavior of two-phase fibrous composites with circular fibers cross-section and nonuniform imperfect contact were analytically investigated via the asymptotic homogenization method (AHM).

In this contribution, the problem of estimating the effective thermal conductivity tensor of two-phase composites made of periodic distributions of unidirectional parallel fibers embedded in a matrix is addressed. Both phases are occupied by linearly-conducting isotropic materials separated by a nonuniform thermal barrier at the interfaces. (In reality, most composites do not have a periodic structure, and inclusions are essentially randomly distributed. However, when the concentration of inclusions is large, the interaction effect becomes dominant and must be accounted for. In this sense, the assumption of periodicity brings these interaction effects into play.) In particular, estimations are provided in the form of variational bounds on the effective thermal energy.

First, local problems obtained from the application of the AHM are shown to be a suitable starting point for the formulation of relevant variational principles. Then, the aforementioned bounds are obtained from a generalization of the Lipton–Vernescu interface comparison method [7], which in turn generalizes the celebrated Hashin–Shtrikman variational principles to the case of uniform interfacial thermal resistance. Also, nontrivial classical-type bounds are obtained.

This article is structured as follows: Section 2 is devoted to the statement of the problem and to the description of the relevant equations related to the mathematical homogenization of two-phase fibrous composite with a nonuniform imperfect interface with rapidly oscillating and periodic coefficients; Section 3 presents the derivation of the variational principles and related classical-type and improved bounds, respectively; Section 4 presents numerical results from computational experiments to compare the direct application of the AHM with the bounds obtained in Section 3; finally, some concluding remarks are given.

In what follows, tensor and vector quantities are typed in boldface.

2. Statement of the problem and homogenization

2.1. Geometrical description

Let $\varepsilon \in \mathbb{R}_+^*$ be a small parameter. Let $\Omega \subset \mathbb{R}^3$ be the domain occupied by a two-phase composite built by identical unidirectional fibers distributed inside a matrix with ε-periodicity and having Lipschitzian boundary $\partial\Omega$. Fibers are oriented in directions parallel to the Ox_3 axis with cross-sections in the Ox_1Ox_2 plane, as illustrated in Fig. 1a. Matrix and fibers occupy subdomains Ω_1^ε and Ω_2^ε, respectively, with $\Omega = \Omega_1^\varepsilon \cup \Omega_2^\varepsilon \cup \Gamma^\varepsilon$, where $\Gamma^\varepsilon = \partial\Omega_2^\varepsilon$ is the interface between Ω_1^ε and Ω_2^ε. Let $Y \subset \mathbb{R}^2$ be the unit cell whose periodic replication generates the cross-section of Ω, which is described in terms of the so-called local variables $y_i = x_i/\varepsilon$, $i = 1, 2$. Then, matrix and fiber occupy subdomains Y_1 and Y_2, respectively, such that $Y = Y_1 \cup Y_2 \cup \Gamma$, where $\Gamma = \partial Y_2$ is the interface, as depicted in Fig. 1b. There, the angle θ of the cell is assumed to remain constant and $\mathbf{n}$ is the outward unit normal vector to Y_1 on Γ. As $|Y| = 1$, the phase concentrations (area fractions) are $c_i = |Y_i|$.