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Stochastic characteristics of powder metallurgy processing

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ABSTRACT

Stochastic characteristics of powder metallurgy processing are analyzed. Theoretical formulation is described which presents probabilistic distributions of material constituents. The described method is based on Poisson processes. Examples of stochastic characteristics of powder metallurgy processing are presented and discussed. The results derived can be used for evaluation of powder metallurgy materials and the subsequent quantitative stochastic design as well as stochastic optimization of powder materials and powder processing.

1. Introduction

New material technologies require the improvement of material characteristics such as wear and corrosion resistance and strength properties. Powder metallurgy (PM) has much to offer because sintering of powder materials produces materials of extremely fine and uniform microstructure and enables the formulation of materials composed from different constituents yielding unique property combinations [1–3]. In the paper, stochastic characteristics of powder metallurgy processing are analyzed. We focus on PM materials, but similar procedure can be applied for powder spray or coating processes. A theoretical formulation is described which presents probabilistic distributions taking into account material constituents Theoretical considerations presented here are based on [4] and [5]. The described method is based on Poisson processes described also in [6,7], for instance. Examples of stochastic characteristics of powder metallurgy processing are discussed. The results derived can be used for evaluation of powder metallurgy materials and the subsequent quantitative stochastic design as well as stochastic optimization. In the author's papers [8–13] some chosen random properties of PM material characteristics were discussed, but they do not concern Poisson processes in PM materials. The use of other statistical representations of powders and granular materials are undertaken for instance in [14–16]. The main novelty of this paper is to show how point processes can be used to evaluate the powder metallurgy processing and powder metallurgy materials.

2. Binomial process in PM materials

Consider a powder material with constituents described by points. Such a simplification can always be done, neglecting the diameter of each constituent. In the paper, it is assumed that by the point we denote the centre of each constituent of powder material. To take a very simple example; consider a fixed number of constituents considered as points at random locations inside a region, Ω, of three-dimensional space. Let $X_1, \ldots, X_n$ be independent random points/constituents which are distributed in Ω. The probability density of each X_i is

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$$f(x) = \begin{cases} 1/\lambda_2(\Omega) & \text{for } x \in \Omega, \\ 0 & \text{for } x \notin \Omega. \end{cases} \quad (1)$$

In expression (1) we denote the area of Ω by $\lambda_2(\Omega)$. A realization of this process is shown in Fig. 1. In this figure an example of a binomial point process in 316L + 0.20% B powder material sintered in hydrogen is shown. In Fig. 2 we present a realization of a binomial point process for the material shown in Fig. 1. In Fig. 3a binomial point process in the ZrO_2/NiCr PM material is shown and its theoretical realization is presented in Fig. 4.

For any bounded set B in R^2 we have

$$P(X_i \in B) = \int_B f(x)dx = \frac{\lambda_2(B \cap \Omega)}{\lambda_2(\Omega)}. \quad (2)$$

The stochastic characteristics of powder material can be described by the variables $N(B)$ and $V(B)$:

$$N(B) = \sum_{i=1}^{n} 1\{X_i \in B\}, \quad (3)$$

$$V(B) = \min_{i=1}^{n} 1\{X_i \notin B\}. \quad (4)$$

We see that $N(B)$ has a binomial distribution with parameters n and $p = \lambda_2(B \cap \Omega)/\lambda_2(\Omega)$. The process is often called the binomial process.

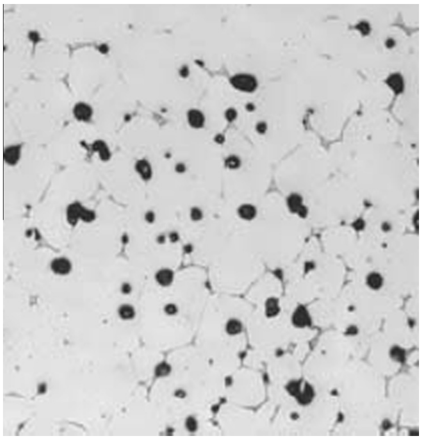

Fig. 1. Binomial point process in 316L + 0.20% B PM material sintered in hydrogen. Photomicrograph from [17].

Fig. 2. Theoretical realization of a binomial point process.