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Formation of sand ripples under a turbulent liquid flow

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ABSTRACT

Sand ripples are commonly observed in both nature and industry. For example, they are found on riverbeds and in oil pipelines that transport sand. In both natural and industrial cases, ripples increase friction between the bed and fluid and are related to flooding, high pressure drops, and transients. Ripples appear when sediments are entrained as bed load (a mobile granular layer) and are usually considered to be the result of initial bedforms that eventually saturate. Given the small aspect ratio of the initial bedforms, linear analyses can be used to understand the formation of ripples. This paper presents a linear stability analysis of a granular bed under a turbulent flow of a liquid. This analysis takes into consideration all the main mechanisms and parameters involved in the turbulent liquid case, including some important parameters that have not yet been considered together such as the bed compactness and the bed-load threshold shear stress. The results of this analysis are compared with published experimental results and they show good agreement.

1. Introduction

Turbulent flows of liquids in the presence of sand are frequent in both nature and industry. Some examples of such flows include river flows, ocean flows, and oil–water–sand flows in petroleum pipelines. Under moderate shear stresses, a granular bed is formed that is not fluidized by the liquid, and the sand is carried as a mobile layer in contact with the fixed part of the sand bed. This mode of transport is known as bed load.

A granular bed entrained as bed load may induce the formation of ripples and dunes [1]. Ripples are bedforms whose wavelengths scale with the grain diameter but not with the flow depth [2,3]. They are usually considered to be a result of initially two-dimensional bedforms that saturate eventually [4,5]. Dunes are bedforms whose wavelengths scale with both the flow field and the flow depth [2,3] and may be considered as the result of coalescence of ripples [6–9]. Ripples and dunes increase friction between the bed and fluid and are related to flooding, high pressure drops, and transients. Although of importance, the formation of aquatic ripples has not yet been completely understood.

Many studies have been conducted in the past decades on the stability of granular beds sheared by fluids, most of them employing linear stability techniques [10–13,3,14–17]. Although this approach has been criticized in the case of aquatic dunes [8,9], it is justified in the case of aquatic ripples given the small aspect ratio of the initial bedforms from which ripples are formed.

More recently, Franklin [4] described the main mechanisms of ripple formation and presented a linear stability analysis for the specific case of turbulent liquid flows far from the threshold for grain displacement [1,2]. The analysis gave expressions for the wavelength, growth rate, and celerity of initial bedforms and demonstrated their variation with fluid stresses, grain diameters, and local slope. However, the analysis neglected the effects of the bed-load threshold shear stress, bed

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compactness, and settling velocity on bed stability. To the best of the author's knowledge, a complete linear analysis that considers all the physical effects (including the threshold effects and bed compactness) has not yet been performed.

Franklin [5] presented a nonlinear stability analysis within the same scope as one of his previous papers [4]. This nonlinear analysis, based on the weakly nonlinear approach [18], showed that the initial bed instabilities saturate, because of which the wavelengths of ripples can be predicted by linear analyses even if ripples observed in nature are in a nonlinear regime.

This paper addresses the formation of sand ripples under turbulent flows of liquids, and presents a linear stability analysis that predicts the growth rate, celerity and length scale of the initial instabilities from which ripples are formed. The present analysis is different from previous works in that it considers all the main physical effects, including the threshold shear stress for grain displacement and bed compactness.

Section 2 describes the physics and main equations involved in the linear stability analysis. Section 3 presents the results of the stability analysis and compares it with published experimental results. Section 4 concludes the paper.

2. Linear stability

Franklin [4] presented a linear stability analysis of a granular bed sheared by a turbulent liquid flow, without free-surface effects. The absence of a free surface is justified in the case of ripples, as these forms do not scale with the flow depth. Franklin's analysis was based on four equations, which describe the mass conservation of granular matter, fluid flow perturbation caused by the bed shape, the transport of granular matter by a fluid flow, and the relaxation effects related to the transport of grains. Although Franklin [4] presented the main mechanisms of ripple formation, he neglected the effects of the threshold shear stress for grain displacement, the bed compactness, and the settling velocity of grains on bed stability.

The linear model presented next is derived from that presented in Franklin [4]. This model is constructed in two dimensions, which is justified by Squire's theorem [19]. The model considers all the main mechanisms and effects involved in ripple formation; therefore, the stability analysis is more comprehensive than previous ones. The main equations, including those of Franklin [4], are presented next for completeness of the paper; their development is reported in detail in Franklin [4].

2.1. Conservation equations

The conservation equations used in this analysis are the mass conservation of granular matter and the momentum balance between the liquid and the grains. The mass conservation of grains in two dimensions relates the local height of the bed, h, to the local transport rate (volumetric) of grains per unit width q:

$$\frac{\partial h}{\partial t} + \frac{1}{\phi}\frac{\partial q}{\partial x} = 0, \tag{1}$$

where t is the time, x is the longitudinal direction and ϕ is the bed compactness. The momentum balance between the liquid and the grains is usually obtained by dimensional analysis, as there is no consensus about the rheology of granular matter. Semi-empirical momentum balances between the liquid and the grains were proposed in the previous decades, and the obtained expressions relate the bed-load transport rate to the shear stress caused by fluid flow on a granular bed. The expression proposed by Meyer-Peter and Müller [20], one of the most frequently used transport rate equations, is based on data from exhaustive experiments, and for this reason, it is used in the present model. The volumetric transport rate of grains per unit width q_0 for a fully developed flow [20] is given by

$$q_0 = D_1(\tau_0 - \tau_{th})^{3/2}, \tag{2}$$

where τ_0 is the shear stress on the granular bed caused by the fully developed flow (the unperturbed, basic state flow described next) and τ_{th} is the threshold shear stress for the incipient motion of grains [1]. D_1 is given by:

$$D_1 = \frac{8}{\rho^{3/2}[(S-1)g]}, \tag{3}$$

where ρ is the specific mass of the liquid, $S = \rho_p/\rho$, ρ_p is the specific mass of the grain material and g is the acceleration of gravity. According to Eq. (3), D_1 is constant for given fluid and grain types.

There is not a universal expression for the bed-load transport rate; on the contrary, there are several empirical and semi-empirical laws, many of them with the same functional form of the Meyer-Peter and Müller [20] equation. Usually, the only difference is a multiplicative prefactor. Because the transport rate equations are semi-empirical laws, experimental uncertainties are included in the multiplicative prefactor. In addition, high uncertainties are present in the determination of the threshold shear stress τ_{th} because the threshold for incipient motion depends on the surface density of the moving grains. Charru et al. [21] showed that the surface density of moving grains decays due to an increase in bed compactness, caused by the rearrangement of grains, known as armoring, which leads to an increase in the threshold shear rate for the bed load. The