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A new scalarization method for finding the efficient frontier in non-convex multi-objective problems

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ABSTRACT

One of the most important issues in multi-objective optimization problems (MOPs) is finding Pareto optimal points on the Pareto frontier. This topic is one of the oldest challenges in science and engineering. Many important problems in engineering need to solve a non-convex multi-objective optimization problem (NMOP) in order to achieve the proper results. Gradient based methods, such as Normal Boundary Intersection (NBI), for solving a MOP require solving at least one optimization problem for each solution point. This method can be computationally expensive with an increase in the number of variables and/or constraints of the optimization problem. Nevertheless, the NBI method is a technique motivated by geometrical intuition to provide a better parameterization of the Pareto set than that provided by other techniques. This parameterization is better in the sense that the points obtained by using the NBI method produce a more even coverage of the Pareto curve and this coverage does not miss the interesting middle part of the Pareto curve.This useful property, provides an incentive to create a new method. The first step in this study is using a modified convex hull of individual minimum (mCHIM) in each iteration. The second step is introducing an efficient scalarization problem in order to find the Pareto points on the Pareto front. It can be shown that the corresponding solutions of the MOP have uniform spread and also weak Pareto optimal points. It is notable that the NBI and proposed methods are independent of the relative scale of different objective functions. However, it is quite possible that obtaining a solution of the NBI method not be Pareto optimal (not even locally). Actually, this method aims at getting boundary points rather than Pareto optimal points that will lead to these points which may or may not be a Pareto optimal point. The effectiveness of this method is demonstrated with various test problems in convex and non-convex MOP cases. After that, a few test instances of the CEC 2009 (Zhang et al. 2008) using the proposed method are studied. Also, the relationship between the optimal solutions of the scalarized problem and the Pareto solutions of the multi-objective optimization problem is presented by several theorems.

1. Introduction

In the present world, people have to deal with urbanization and industrialization, increase of water and energy demands, environmental pollution, shortage of natural resources and food, and many other challenges that necessitate the

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development of a multidisciplinary approach for analyzing divers mechanisms and consequences of modern civilization. Multi-criteria decision making (MCDM) is concerned with theory and methodology that can treat complex problems encountered in business, engineering and other areas of human activities (see e.g., [1–4]). There are usually multiple conflicting objectives in scientific problems (see e.g., [4–8]). Since the objectives are in conflict with each other, there is a set of Pareto optimal solutions. In other words, the optimal decision needs to be made by identifying the best trade-offs among these criteria, which is the goal of multi-objective optimization. The area of MCDM has developed rapidly, as demonstrated by Steuer et al., [9]. Researchers have studied multi-objective optimization problems from different perspectives, and thus there exist different solution philosophies and goals to solve them. One might be to find a set of Pareto optimal solutions, and/or quantifying the trade-offs which can be satisfied in objectives and/or finding a solution that satisfies the necessities of a human decision maker (DM). Usually it is not economical to generate the entire Pareto surface, due to the high computational cost for function evaluations. In engineering problems, one of the aims is to find a representative sample of Pareto optimal points. There are usually multiple Pareto optimal solutions for multi-objective optimization problems. This means that solving such a problem is not as straightforward as it is for a conventional single-objective optimization problem. Therefore, different researchers have defined the term solving a multi-objective optimization problem in various ways. Some methods convert the original problem with multiple objectives into a single-objective optimization problem. This is called a scalarized problem, although many methods act in other ways. If scalarization is done carefully, Pareto optimality of the solutions obtained can be guaranteed. There has been a great deal of effort by the researchers in the area (especially in recent years) for developing methods to generate an approximation of the Pareto front (see e.g., [10–26]).

Multi-objective optimization methods can be divided into four classes[27]. In so-called no preference methods, no DM is expected to be available, but neutral compromise solutions are identified without preference information. The other classes are so-called a priori, a posteriori and interactive methods that all involve preference information from the DM in different ways. In a priori methods, preference information is first asked from the DM and then a solution best satisfying these preferences is found. In a posteriori methods, a representative set of Pareto optimal solutions are first found and then DM must choose one of them. In interactive methods, the decision maker is allowed to iteratively search for the most preferred solution. In each iteration of the interactive method, the DM is shown Pareto optimal solution(s) and describes how the solution(s) can be improved. Scalarizing a multi-objective optimization problem means formulating a single-objective optimization problem such that optimal solutions to the single-objective optimization problem are Pareto optimal solutions to the multi-objective optimization problems [27]. In addition, it is often required that every Pareto optimal solution can be reached with some parameters of the scalarization [27]. With different parameters for the scalarization, different Pareto optimal solutions are produced. A general formulation for a scalarization of a multi-objective optimization is:

$$\begin{array}{ll} \text{minimize} & g(f_1(\mathbf{x}),\ldots,f_p(\mathbf{x}),\theta) \\ \text{subject to} & \mathbf{x} \in X_\theta, \end{array} \tag{1}$$

where θ is vector parameter, the set $X_\theta \subseteq X$ is a set depending on the parameter θ and $g : \mathbf{R}^{p+1} \to \mathbf{R}$ is a function.

One of these methods is the weighted sum method (WSM) [1,28] which is also called linear scalarization. The idea of the WSM is the conversion of the MOP into a single objective optimization problem using a convex combination of objectives. Even though under some conditions, the solution obtained by the WSM method is a Pareto optimal point, the WSM method cannot generate any points in the non-convex part of the Pareto front and also, the WSM may duplicate solutions with different weight combinations. However, this method does not often produce an even distribution of Pareto points [29]. On the other hand, a posteriori method aims at producing all the Pareto solutions or a representative subset of the Pareto optimal solutions. Most a posteriori methods fall into either one of the following two classes: mathematical programming based a posteriori methods, where an algorithm is repeated and each run of the algorithm produces one Pareto optimal solution, and evolutionary algorithms where one run of the algorithm produces a set of Pareto optimal solutions.

Well-known examples of mathematical programming based a posteriori methods are the Normal Boundary Intersection (NBI) [11], Modified Normal Boundary Intersection (NBIm) [30], Normal Constraint (NC) [31,32], Successive Pareto Optimization (SPO) [33] and Directed Search Domain (DSD) [34] methods that solve the multi-objective optimization problem by constructing several scalarizations. The solution to each scalarization yields a Pareto optimal solution, whether locally or globally. Cohon [35] developed a constraint method (CM) and Weck [36] developed an adaptive weighted sum method (AWS) for multi-objective optimization. The scalarizations of the NBI, NBIm, NC, DSD, CM and AWS methods are constructed with the target of obtaining evenly distributed Pareto points that give a good evenly distributed approximation of the real set of Pareto points.

The NBI method [11] uses a series of tractile single objective problems to approximate the Pareto front. Starting from a point on the Utopia plane which passes through individual function minimizers, a single objective optimization problem in NBI is to maximize the distance from the starting point to a point located on the normal line of the Utopia plane. With different starting points on the Utopia plane, NBI produces well distributed solutions. However, the NBI and its improvements such as the Normal Constraint (NC) and modified normal boundary intersection methods (mNBI) are alternatives not affected by these problems and do not sacrifice computational time to better obtain the Pareto frontier. The proposed modification which will be based on the NBI method with some fundamental changes is more suitable for engineering design and the other nonlinear multi-objective optimization problems. For example, the proposed method provides a gradient based algorithm which, in the case of continuous Pareto frontier for bi-objective optimization problems, obtains the Pareto frontier