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Thermo-viscoelastic materials with fractional relaxation operators

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ABSTRACT

The new model of linear thermo-viscoelasticity for isotropic media taking into consideration the rheological properties of the volume with fractional relaxation operators is given. The governing equations are taken in a unified system from which some essential theorems on the linear coupled and generalized theories of thermo-viscoelasticity can be easily obtained. The resulting formulation is applied to several concrete problems, a thermal shock problem and a problem for a half-space subjected to ramp-type heating as well as a problem of a layer media. Laplace transform techniques are used. According to the numerical results and its graphs, conclusion about the new theory has been constructed. Some comparisons have been shown in figures to estimate the effect of fractional relaxation operators and ramping parameter of heating with different theories of thermoelasticity.

1. Introduction

It is well known that the usual theory of heat conduction based on Fourier's law predicts infinite heat propagation speed. It is also known that heat transmission at low temperature propagates by means of waves. These aspects have caused intense activity in the field of heat propagation. Extensive reviews on the second sound theories (hyperbolic heat conduction) are given in Chandrasekharaiah [1] or Hetnarski and Ignaczak [2].

Due to the recent large-scale development and utilization of polymers and composite materials, the linear-viscoelasticity remains an important area of research.

Linear viscoelastic materials are rheological materials that exhibit time temperature rate-of-loading dependence. When their response is not only a function of the current input, but also of the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. A general overview of time-dependent material properties has been presented by Tschoegl [3]. The mechanical-model representation of linear viscoelastic behavior results was investigated by Gross [4]. One can refer to Atkinson and Craster [5] for a review of fracture mechanics and generalizations to the viscoelastic materials, and Rajagopal and Saccomandi [6], for non-linear theory.

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Nomenclature

λ, μ	Lame' constants
C_E	specific heat at constant strains
K	$\lambda + (2/3)\mu$, bulk modulus
C_o^2	$\frac{K}{\rho}$, longitudinal wave speed
ε_{ij}	components of strain tensor
e_{ij}	components of strain deviator tensor
σ_{ij}	components of stress tensor
S_{ij}	components of stress deviator tensor
e	ε_{ii}, Dilatation
E	modulus of elasticity
k	thermal conductivity
$R(t, \beta)$	relaxation functions
T	absolute temperature
u_i	components of displacement vector
α_T	coefficient of linear thermal expansion
γ	$3K\alpha_T$
δ_{ij}	Kronecker's delta.
T_o	reference temperature
v	Poisson's ratio
ε	$\frac{\gamma^2 T_0}{k\eta_o \rho C_0^2}$, Thermal coupling parameter
Θ	$T - T_0$, such that $\lvert\Theta/T_0\rvert \ll 1$
Q	strength of applied heat source per unit mass
ρ	mass density
τ	the ratio of the shear viscosity to Young's modulus
τ_o, υ	two relaxation times
α, β	fractional orders
t	time
$\Gamma(.)$	Gamma function

Within the theoretical contributions to the generalized thermoviscoelasticity theory are the proofs of uniqueness theorems under different conditions by Ezzat and El Karamany [7,8] and the boundary element formulation was done by El-Karamany and Ezzat [9,10].

The model of the equation of generalized thermo-viscoelasticity, ignoring the relaxation effects of the volume, with one relaxation time and with two relaxation times are established by Ezzat et al. [11–13]. Ezzat [14] investigated the relaxation effects on the volume properties of an electrically conducting viscoelastic material. Recently, Ezzat et al. [15] established the equations of the linear theory of generalized thermo-viscoelasticity for an electrically conducting isotropic media permeated by a primary uniform magnetic field, taking into account the rheological properties of the volume.

Fractional calculus has been used successfully to modify many existing models of physical processes. Caputo and Mainardi [16] and Caputo [17] found good agreement with experimental results when using fractional derivatives for description of viscoelastic materials and established the connection between fractional derivatives and the theory of linear viscoelasticity. Adolfsson et al. [18] constructed a newer fractional order model of viscoelasticity.

Recently, Ezzat [19–21] established a new model of fractional heat conduction equation using the new Taylor series expansion of time-fractional order which developed by Jumarie [22]. Ezzat and El-Karamany [23] studied a problem of thermo-viscoelastic for a perfect conducting half-space in the context of fractional magneto-generalized thermoelasticity. Povistinko [24] obtained the fundamental solutions to time-fractional heat conduction equations in two joint half-lines. Ezzat et al. [25] derived a new fractional relaxation operator using the methodology of fractional calculus. Abbasat el. [26] solved thermoelastic interactions problems in anisotropic media in the context of the theory of fractional order generalized thermoelasticity. Sherief and Abd El-Latief applied the fractional order theory of thermoelasticity to a 1D thermal shock problem for a half-space [27]. A state-space method for the calculation of dynamic response of systems made of viscoelastic materials with exponential type relaxation kernels was introduced by Menon and Tang [28]. Extension of thermo-viscoelastic and magneto-thermo-viscoelastic problems in generalized theory are found to be present in the works of many researchers out of which Mukhopadhyay and Bera [29], Ezzat [30]and El-Karamany and Ezzat [31].

This article introduces a new model for the linear theory of generalized thermo-viscoelasticity, taking into consideration the fractional relaxation operators effects of the volume. The resulting formulation is applied to several concrete one-dimensional problems. The formulation is applied to the generalized thermoelasticity theories: Lord–Shulman [32], Green–Lindsay [33] and to the dynamic coupled theory, Biot [34].