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Scaled boundary finite-element method for solving non-homogeneous anisotropic heat conduction problems

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ABSTRACT

In this study, we employ the scaled boundary finite-element method (SBFEM) to solve two-dimensional heat conduction problems. In the last two decades, SBFEM has been applied successfully in different fields of engineering and science. SBFEM utilizes the advantages of both the boundary element and finite-element methods. As found in the boundary element method, only the boundary is discretized to reduce the special discretization by one. Unlike the finite-element method, no fundamental solution is needed. Using a scaling center, the geometry of the problems is transformed into scaled boundary coordinates, including the radial and circumferential coordinates. The boundary of the problem represents the whole computational domain. The finite-element approximation of the circumferential coordinates yields the analytical equation in the radial coordinate. In the steady-state analysis, an eigenvalue problem is solved to compute the stiffness matrix and to obtain the temperature field in the domain. In the transient analysis, the stiffness matrix determined from the eigenvalue problem as well as the mass matrix determined from the low frequency behavior forms a system of first-order ordinary differential equations, which need to be solved for the temperature field using time integration schemes. We outline the formulation and solution procedure for the SBFEM when modeling heat conduction problems. We developed a code in MATLAB to obtain the numerical solution. Several numerical examples are presented to demonstrate the simplicity, versatility, and applicability of the SBFEM during the simulation of heat conduction problems with non-homogeneous and anisotropic media. Complex irregular geometries are modeled. An example with flux singularity is numerically simulated to further demonstrate the advantages of this technique. The method is validated by comparing the SBFEM results with those obtained using FEM and those from previous studies.

1. Introduction

Two-dimensional (2D) parabolic equations play very important roles in many engineering applications. The analysis of seepage in porous media and the temperature distribution among solids are two of the most important applications.

Because of its considerable importance in many engineering applications, the fundamentals of heat transfer in solid particles have long attracted the attention of researchers. Many practical heat conduction questions lead to problems that are not solvable readily by classical methods, such as the separation of variables techniques or Green's functions. However,

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problems with only simple geometry and material properties can be solved using analytical techniques. Thus, engineers must use approximate numerical approaches in practice to deal with complex geometry and material properties. The finite difference method (FDM) is often used as a conventional tool for studying heat conduction problems. The FDM is a mesh-dependent method where a boundary-fitted mesh is employed. Therefore, advanced and sophisticated techniques need to be employed for discretizing complex geometries. The most popular numerical tool in engineering practice is the finite-element method (FEM). FEM has been applied successfully to heat conduction problems. When dealing with infinite domains, it needs to truncate the domain, thereby resulting in a reduction in accuracy. However, infinite elements coupled with finite elements have been employed widely in previous studies for capturing unbounded domains. In addition to numerical schemes, the boundary element method (BEM) based on integral equations has also attracted the attention of researchers for solving the partial differential governing equations. In the BEM, only the boundary is discretized and the complex geometries can be discretized easily. However, the complexity of the fundamental solutions needed in modeling increases considerably for general anisotropic materials, thereby limiting the application of the BEM. Many researchers have used these three popular numerical schemes to solve 2D heat conduction problems. In the following, we present a short review of these applications.

Wang et al. [1] used the finite-volume method for cylindrical heat conduction problems. A FEM/FDM scheme was proposed by Wang et al. [2] for non-classical heat conduction problems. Pulvirenti et al. [3] studied FDM solutions of hyperbolic heat conduction with temperature-dependent properties. Wang [4] solved transient one-dimensional heat conduction problems by FEM. Desilva [5] presented a coupled BEM and FDM for the heat conduction in laser processing. Kutanaei et al. [6] investigated the mesh-free modeling of 2D heat conduction between eccentric circular cylinders. A meshless method based on the local weak-forms was applied by Wu and Tao [7] to steady-state heat conduction problems. Gao and Wang [8] used the interface integral BEM for solving multi-media heat conduction problems. Zhang et al. [9] employed a novel BEM for solving anisotropic potential problems. The local radial basis function-based differential quadrature (RBF-DQ) method [10] has been used for solving 2D heat conduction problems.

In this study, we apply the semi-analytical technique of SBFEM directly to heat conduction problems in two- or three-space variables. We only address the formulation and solution procedures for 2D problems in this study.

The scaled boundary finite-element method (SBFEM) was developed in the 1990s based on the similarity concept for the dynamic stiffness of unbounded domains. The original formulation and its application to 2D and three-dimensional (3D) vector waves and fracture mechanics were presented by Wolf and Song [11]. Later, a new derivation was derived, which is consistent with the FEM based on Galerkin's weighted residual method [12]. Deeks and Wolf [13] employed the principle of virtual work to re-derive the scaled boundary finite-element equations. Analytical solutions were derived in [14] for problems with certain body loads, such as concentrated loads and loads that vary as power functions of radial coordinates. The SBFEM was extended to model the time-harmonic and transient responses of non-homogeneous elastic unbounded domains by Bazyar and Song [15,16]. New developments were made to enhance the solution procedures for modeling unbounded domains in both the frequency and time domains [17,18]. Steady-state confined and unconfined seepage problems were addressed in [19,20], and transient seepage was considered in [21].

The remainder of this paper is organized as follows. The governing differential equations of the heat conduction problems are given in Section 2. A brief summary of the SBFEM concept is provided in Section 3. The formulation of the SBFEM for heat conduction problems is explained in Section 4. Section 5 provides the solution procedures of the SBFEM for transient heat conduction problems. Five numerical examples are solved in Section 4 to evaluate the performance of the proposed scheme. Finally, we give our conclusions in Section 5.

2. Governing equations

The governing equations for 2D heat conduction problems are described in this section. In a heat conduction problem, a solid Ω experiences thermal loads and we need to determine the temperature field in this solid when it is in equilibrium with the external environment. The equilibrium state should correspond to the transient state. In addition, our study is restricted to the linear case where the thermal conductivity is independent of temperature.

The governing equations for 2D transient heat conduction problems are written as:

$$\{L\}^T\{q\} + \mu\dot{T} = 0, \tag{1}$$

where the material constant μ is the product of the mass density ρ and the specific heat c_v of the material, and $\{q\}$ is the heat flux vector related to the temperature field T defined as

$$\{q\} = -[k]\{L\}T, \tag{2}$$

where $[k] = \begin{bmatrix} k_x & k_{xy} \\ k_{xy} & k_y \end{bmatrix}$ is the anisotropic thermal conductivity matrix and $\{L\}$ is the gradient operator

$$\{L\}^T = \left\{ \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right\}. \tag{3}$$

Eq. (1) is transformed into the frequency domain by applying the Fourier transform