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Analytical solutions of stress singularities in a sectorial plate based on Reissner–Mindlin plate theory

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A R T I C L E I N F O

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A B S T R A C T

This paper aims at the stress singularities in a sectorial plate based on Reissner–Mindlin plate theory. The Mellin transform is applied to solve the governing equations of the plate. Both the bending stress singularities due to concentrated moments at boundaries and transverse shear stress singularities due to concentrated transverse shear forces at boundaries are investigated. The application of inverse Mellin transform gives the exact forms of singular stress fields near the corner, including the stress singularity orders and the generalized stress intensity factors. For various plate angles and loading conditions, the singular stress parameters are computed by finite element models and the results are compared to the analytical solutions. The comparisons reveal that the stresses obtained by the analytical procedure proposed in this paper are correct.

1. Introduction

Stress singularities in elasticity arise due to the geometric discontinuities such as sharp corners in plates with V-notches. The study to the singular behavior helps the engineer to understand the fracture phenomenon of a structure [1]. In 1952, Williams [2] first introduced the eigenfunction expansion method to analyze the stress singularity in a plate with V-notch subjected to inplane loading. The stress singularity orders can be determined by solving the characteristic equations. Later some pioneers also made efforts on similar problems. For example, some reports extended the problems to anisotropic material and multi-material structures [3–6]. Recent progresses of stress singularity have been extended to piezoelectric materials [7] and magneto-electro-elastic materials [8]. Their studies showed that the eigenfunction expansion method is a powerful tool to solve the stress singularity orders under various types of problems.

For plates with V-notches subjected to bending moments and transverse shear forces, the stresses at the tips of the notches also become singular, same as the case of inplane loadings. For plate bending, Williams used the classical plate theory in conjunction with the eigenfunction expansion method to obtain the characteristic equations for various boundary conditions [9]. Burton and Sinclair [10] introduced stress potential to solve the thick plate bending problem. They followed Williams' procedure to determine the stress singularity order. Huang [11] investigated the singularities at the corner of a thick plate based on Reissner–Mindlin plate theory. Both the moment and transverse shear force singularities are considered. Later Huang [12] investigated stress singularities in a plate based on higher-order shear deformation theory. For vibration of a plate with a V-notch, the natural frequency can be determined by Ritz method. It has been proven that with the aid of the asymptotic fields near the corner tip, the convergence speed in determining the natural frequency is much faster [13,14].

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In the eigenfunction expansion method, the stresses are assumed to be as forms of separation of variable, i.e., $\sigma_{ij}(r,\theta) = K\Theta_{ij}(\theta)r^{\lambda}$, where $\Theta_{ij}(\theta)$ is the angular function, λ is the eigenvalue, and the multiplication K denotes the generalized stress intensity factor. By considering the tractions free at the boundaries of the V-notched plate in conjunction with the assumed forms of the stresses, an eigenvalue problem can be derived. The eigenvalue λ is then determined and equivalent to the stress singularity order if the singular stresses occur. The angular function can be determined by the eigenvector. This method provides an efficient way to determine the stress singularity order and the associated angular function. However, for non-zero tractions or non-zero forces applied at boundaries, the generalized stress intensity factor K cannot be determined by the eigenfunction expansion method.

The Mellin transform is a technique to determine the stress intensities near the corner tip. Bogy [3,4] first applied this technique to solve the asymptotic stress fields near the corner tip of a plate under inplane loading. In the past two decades, the Mellin transform has been widely used to solve notched plates under inplane loadings for both isotropic and orthotropic materials [7,15–17]. Unlike the eigenfunction expansion method, the non-zero loadings at boundaries are considered in the method of Mellin transform. The transformed stresses and displacements can be determined under a set of non-homogeneous boundary conditions. The stresses fields as well as the generalized stress intensity factors can be determined by applying the inverse transform. To the best of the author's knowledge, the Mellin transform has not been applied to the plate under bending moments and transverse shear forces.

In this paper, the singular stress fields of a sectorial thick plate with infinite radius are investigated based on the Reissner–Mindlin plate theory and Mellin transform technique. The plate is subjected to moments or transverse shear forces at boundaries. The asymptotic solutions near $r = 0$ of the transformed governing equations of the thick plate are obtained. The integral path of the inverse transform is discussed and the residue theorem is used to calculate the integral so that the asymptotic stress fields, including the stress singularity order and stress intensity, near the corner are obtained in exact forms. Both the bending stress singularities and transverse shear stress singularities are investigated. The finite element models of various plate angle α and loading conditions are investigated to verify the analytical solutions.

2. Basic equations in Reissner–Mindlin plate theory

Consider a sectorial plate with plate angle α and infinite radius subjected to moments and shear forces at boundaries as shown in Fig. 1. The thickness of the plate is t and a cylindrical coordinate system (r, θ, z) is used in the following derivation. The origin of the coordinate is placed at the apex of the corner on the middle plane. Assume that the plate follows the Reissner–Mindlin plate theory, that is, the first order shear deformation theory. The equilibrium equations in polar coordinate for the Reissner–Mindlin plate are given as

$$\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial \theta} + \frac{M_r - M_\theta}{r} - Q_r = 0, \tag{1a}$$

$$\frac{\partial M_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial \theta} + \frac{2M_{r\theta}}{r} - Q_\theta = 0, \tag{1b}$$

$$\frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial \theta} = 0, \tag{1c}$$

where M_r, M_θ and $M_{r\theta}$ are bending moments and Q_r and Q_θ are shear forces. By introducing three displacement and rotation potentials w, ψ_r, and ψ_θ, the moments, shear forces and stresses can be represented as

$$M_r = -D\left[\frac{\partial \psi_r}{\partial r} + \frac{\nu}{r}\left(\psi_r + \frac{\partial \psi_\theta}{\partial \theta}\right)\right], \tag{2a}$$

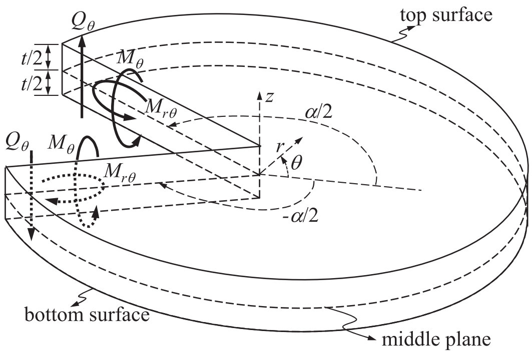

Fig. 1. A sectorial thick plate subjected to bending moments and transverse shear forces at boundaries.