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Short communication

Finite-time mixed outer synchronization of complex networks with time-varying delay and unknown parameters ☆

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ABSTRACT

In this paper, the finite-time mixed outer synchronization (FMOS) of chaotic neural networks with time-varying delay and unknown parameters is investigated. By adjusting control strengths with the updated laws, we achieve the finite-time mixed outer synchronization between two complex networks based on the finite-time stability theory and linear matrix inequality. Furthermore, the unknown parameters estimation of the networks is identified in a finite time. Finally, numerical simulations are given to demonstrate the effectiveness of the analytical results obtained here.

1. Introduction

Chaos synchronization has a great number of applications in many fields from meteorology to Chemical reactions [1], from circadian rhythms in unicellular to multi-cellar organisms [2], etc. Many authors have studied the synchronization problem of some complex networks [3–6]. A wide variety of approaches have been proposed for the synchronization of chaotic systems, including adaptive control [7–10], nonlinear feedback control [11,12], sliding mode control [13,14], observer-based control [15,16], descriptor model transformation method [17], etc.

In recent years, several different types of mixed outer synchronization have been studied, including complete outer synchronization (COS) [18], inverse outer synchronization (IOS) [19], adaptive outer synchronization (AOS) [20], and generalized outer synchronization (GOS) [21]. Li et al. [22] studied the outer synchronization of two complex networks with known parameters through the open-plus-closed-loop control techniques. In [23], the problem of mixed outer synchronization between two complex networks with the same topological structure and time-varying delay was investigated by designing a novel non-fragile linear state feedback controller. After, by using the same controller, He et al. [24] further studied that complex networks with known parameters and coupling time-varying delay could realize finite-time mixed outer synchronization (FMOS) with respect to some constants under suitable constraint. Xu et al. [25] considered the two neural networks

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with bidirectional coupling by the adaptation of control strengths r_i and k_i such that the error systems converged asymptotically to zero.

Up to now, few researches related to mixed outer synchronization and identification of unknown parameters in a finite time have been done. In this paper, based on Lyapunov stability theory and linear matrix inequality, we achieve finite-time mixed outer synchronization for the complex networks with unknown parameters by designing feedback controlling laws. From a practical point of view, it has more advantages to synchronize the chaotic networks in a given time. The results obtained in this paper have optimality in the convergence time of the mixed outer synchronization and topology identification.

This paper is organized as follows. In Section 2, an uncertain general dynamical system is introduced and some mathematical preliminaries used in this paper are given. In Section 3, finite-time mixed outer synchronization of two time-varying delayed neural networks with bidirectional coupling is studied by using the updated laws of coupling strengths. In Section 4, numerical examples are given for supporting the analytical results obtained here. Finally, some conclusions are drawn in Section 5.

2. Preliminaries

Consider the following uncertain dynamical systems

$$\dot{x}_i(t) = G(x_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma x_j(t-\tau(t)), \quad i = 1,2,\ldots,N, \tag{1}$$

where $x_i(t) = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathrm{R}^n$ are the state variables of the ith node. $G : \mathrm{R}^n \to \mathrm{R}^n$ is a continuous nonlinear function. $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ is a positive definite diagonal matrix determining the interaction of variables, $\tau(t) \geqslant 0$ is the time-varying delay, and $\varepsilon > 0$ is the coupling strength. The matrix $C = (c_{ij})_{N\times N}$ is an unknown delayed weight coupling matrix representing the topological structure of the network, if there is a connection from node j to $i, c_{ij} \neq 0$ $(j \neq i)$; otherwise, $c_{ij} = 0$ $(j \neq i)$.

Furthermore, we decompose the nonlinear function $G(x_i(t))$ into three parts $A_d x_i(t) + A_n x_i(t) + F(x_i(t))$, where A_d is the unknown diagonal matrix and A_n is the known non-diagonal matrix, and $A_d = \mathrm{diag}\{\theta_1, \theta_2, \ldots, \theta_n\}$. $F : \mathrm{R}^n \to \mathrm{R}^n$ is a continuous nonlinear function satisfying $HF(x) = F(Hx)$ and $H = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\}, \alpha_l \in \{1, -1, 0\}, l = 1, 2, \ldots, n.$

In the following, we present a Definition, an Assumption and some Lemmas.

Definition 1. The mixed outer synchronization errors are defined as

$$e_i(t) = y_i(t) - Hx_i(t), \quad i = 1,2,\ldots,N, \tag{2}$$

where the scaling matrix $H = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\}, \alpha_l \in \{1, -1, 0\}$ is predefined.

Remark 1. It is easy to see that H includes the known notions of inverse outer synchronization (IOS), complete outer synchronization (COS), and mixed outer synchronization (MOS) as its special cases, when all $\alpha_l = -1,\ \alpha_l = 1,\ \alpha_l = \pm 1\ (l = 1,2,\ldots,n)$.

Assumption 1. The nonlinear function $F(x)$ satisfies that there exists a constant $L > 0$ such that

$$\|F(y) - F(x)\| \leqslant L\|y - x\|, \text{ for all } x, y \in \mathrm{R}^n. \tag{3}$$

Lemma 1 [26]. *Assume that a differential, positive-definite function $V(t)$ satisfies the following inequality:*

$$\dot{V}(t) \leqslant -\beta V^{\eta}(t), \quad \forall t \geqslant t_0, \quad V(t_0) \geqslant 0, \tag{4}$$

where $\beta > 0,\ 0 < \eta < 1$ are two constants. Then, for any given $t_0,\ V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - \beta(1-\eta)(t - t_0), \quad t_0 \leqslant t \leqslant t_1, \tag{5}$$

and

$$V(t) \equiv 0, \quad \forall t \geqslant t_1, \tag{6}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\beta(1-\eta)}. \tag{7}$$