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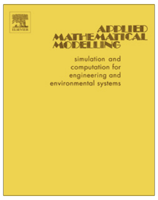

Frequency dependent impedance model for heteropolar magnetic bearings

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A B S T R A C T

Accurate analytical models are fundamental to design and optimise high performance magnetic bearings. Understanding the bearing's dynamic behaviour over its operational envelope is essential to this task and is performed through frequency response analyses. This work formulates an integrated frequency dependent model of a radial heteropolar active magnetic bearing (AMB) in which terms for the stray capacitance, winding resistance, and winding leakage inductance are included. The model is based on a lumped parameter model frequently used for high frequency laminated inductor applications. Two magnetic core models are evaluated. The first comprises a reluctance network approach and incorporates eddy-current, fringing, and leakage correction terms, while the second constitutes a single gapless magnetic core of equivalent length and effective permeability. The accuracy of the latter is improved by intuitively adjusting two parameters in the analytical model. The magnetic bearing parameters calculated as functions of frequency are compared with practical measurements. The integrated single core model demonstrates superior correlation with experimental results for frequencies up to 1 MHz.

1. Introduction

Active magnetic bearings (AMBs) provide frictionless suspension of the rotor through magnetic forces, rendering them key technology for high speed rotating applications. Accurate analytical models provide valuable information regarding the bearing's dynamic behaviour over its operational envelope and are therefore fundamental to optimise bearing performance. This is especially important during the design phase of a high bandwidth AMB to estimate losses in the magnetic material due to eddy currents (induced by the switching ripple). Of particular interest is the frequency response of the AMB which is examined to characterise the variation in operating parameters for different frequency ranges.

Fig. 1(a) displays a frequency dependent lumped parameter model proposed by [1] to describe the frequency response of a laminated heteropolar AMB. The equivalent series resistance and inductance are firstly obtained from a reluctance network model that uses a complex material permeability term. The latter is used to model the eddy current effects in the lamination material which is the main contributor to the core losses [2]. The resulting inductance is complex, where the real term of the reactance denotes the series equivalent core resistance, and the imaginary term the reactance as a result of inductance. Fringing and leakage effects are compensated for by adding a constant leakage reluctance path. Accurate values are determined for the air gap- and leakage-reluctances using finite element methods (FEM). This model displays good correlation

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with experimental results for frequencies up to a few kHz. At high frequencies, the results do not correlate well, which is partly attributed to unmodelled parasitic capacitances [3] and wire resistance.

An equivalent lumped RLC circuit, which is frequently used to model the behaviour of simple laminated iron core inductors with air gaps, is shown in Fig. 1(b). The analytical model proposed by [4] calculates the total parasitic capacitance using the measured first self-resonant frequency which is assumed to be constant. The model shows sufficiently accurate results from zero to above the first self-resonant frequency of the examined inductors. However, the effect of the overall stray capacitance becomes significant in different frequency ranges (i.e. results do not correlate) and dominant at frequencies above the first self-resonant frequency.

This paper extends the work presented in [1,4] and addresses the aforementioned problems via integrated models based on Fig. 1(b) to describe the high frequency behaviour of a complex radial 8-pole heteropolar AMB. The stray capacitance of the winding is derived from an analytical model based on the physical structure of the winding [3]. The equivalent series resistance R_{ac} and inductance L_{ac} are frequency dependent components due to skin and proximity effects in the windings and eddy currents in the laminated iron core [4]. The winding resistance R_w is obtained from equations proposed by [5]. Two methods of determining the main inductance and the series core resistance are examined. First, the reluctance network model proposed by [1] is utilised, and secondly, a modelling technique proposed by [4] is adopted. The latter technique models the complex magnetic structure as a single magnetic core with no air gap and a new equivalent material permeability. The lumped RLC circuit's complex impedance is represented in terms of the equivalent series resistance R_s and reactance X_s depicted in Fig. 1(c). These parameters calculated as functions of frequency are compared with practical measurements.

The contents of the paper are organised as follows. Section 2 gives an overview of the winding self-capacitance model. The winding resistance and leakage inductance models are outlined in Section 3. Sections 4 and 5 report the underlying modelling principles of the two main inductance and series resistance models respectively. The modelled results are compared with experimental measurements for three different cases in Section 6. Some concluding remarks are presented in Section 7.

2. Winding self-capacitance model

The upper operating frequency of inductors is limited by its self-capacitance [3]. Therefore, characterisation of the stray capacitance between the coil windings is important to accurately predict the frequency response of the AMB. The stray capacitance of single-or multiple-layer coils may be determined experimentally, but most methods rely on practical data. In [3], a simple expression is derived to predict the total distributed stray capacitance of inductors based on the physical structure of the windings (i.e. geometry and number of layers).

The distributed stray capacitance is modelled by a single lumped parameter connected between the winding terminals as shown in Fig. 1(b). The total stray capacitance consists of the following components [3]:

- Turn-to-turn capacitance between turns of the same layer;
- Turn-to-turn capacitance between turns of adjacent layers; and
- Turn-to-core and turn-to-shield capacitances.

Fig. 2 shows the basic cell *abcd* representing the turn-to-turn capacitance of a uniformly wound coil. From the figure, it can be seen that the winding geometry is symmetrical. As a first-order approximation, all the basic cells, which include two adjacent turns of the same and different layers, as well as cells adjacent to the core and shield, are considered identical [3]. The lines of the electric field **E** cross the basic cell through the two insulating coatings and the air gap between them. Therefore, a series combination of three capacitors with uniform dielectric material constitutes the elementary capacitance between adjacent turns.

The series combination of the elementary capacitances for the insulating coatings and the air gap, integrated over the basic cell, gives the overall turn-to-turn capacitance [3]

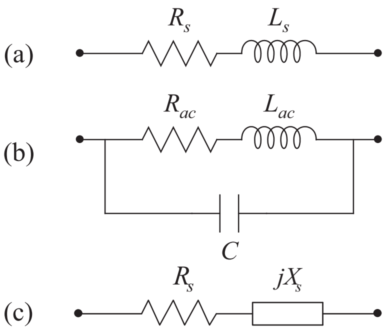

Fig. 1. Equivalent lumped parameter circuits for an inductor: (a) series model (measured by most impedance metres); (b) RLC model; (c) series equivalent model [4].