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# Operator splitting technique with FORCE scheme employed to simulate pressure wave motion inside gun chamber

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The work is focused to study the mathematical modeling and numerical simulation of solid propellant combustion during internal ballistics cycle. Here, we have used a reduced single phase model from Baer–Nunziato two-phase flow model for gas and solid mixture of propellant. The model consists of balance equations of mass, momentum and energy with constitutive laws. First ORder CEntred (FORCE) scheme is employed to provide an insight of motion of pressure wave in gun chamber during burning of igniter and solid propellant. The study is performed with the constant emission rate of igniter gases. The burning rate of propellant is expressed as a non-linear function of pressure in terms of pressure exponent. Depending on the pressure exponent three cases are evolved: constant, linear and non-linear rate of burning of propellant. FORCE scheme captures the motion of pressure wave with time and space. The local phenomena of pressure, temperature, velocity, density and rate of burning are demonstrated from breech end to the base of the projectile at different time. The effect of discontinuous pressure on burning dimensions of grain are examined. The non-uniform burning of grain dimensions may result into grain fracture. This study helps in designing the system to avoid such situations.



## 1. Introduction

In-depth analysis of ballistic cycle is of the prime importance for designing the gun. The ballistic cycle involves processes of igniter and solid propellant ignition, flame spreading and projectile motion. Igniter releases hot gases into the chamber, causes burning of solid propellant. The hot gases from igniter and propellant increases the pressure and temperature in the chamber. The pressure in the chamber acts on the projectile base and cause motion of projectile in the gun barrel. Ballistic cycle is completed when the projectile exits from the muzzle. The whole process is complete in few milliseconds and it is highly complex. Due to the rapid processes and complexity, experimental study is difficult and also expensive. Mathematical modeling and simulation approach is a good option and can be applied for complete analysis of physical processes occurring in the ballistic cycle.

Mathematical modeling approach for combustion studies inside a gun has been a topic of study for several decades. Four different types of theoretical models to study the propellant combustion are available in the literature [1–3], viz. (i) statistical models, (ii) continuum-mechanics models, (iii) formal averaging models and (iv) two-phase fluid dynamics models. There are several numerical schemes that can be employed to solve these models. Sheu [4] had studied the two-phase flow

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**Nomenclature**

| | |
|---|---|
| $\rho$ | density [kg/m$^3$] |
| $u$ | velocity of gas [m/s] |
| $E$ | total energy |
| $p$ | pressure [pa] |
| $e$ | internal energy |
| $\dot{m}_{ign}$ | mass transfer rate of igniter [kg/m$^3$/s] |
| $\dot{m}$ | mass transfer rate of solid propellant [kg/m$^3$/s] |
| $Q_{ign}$ | heat energy released from igniter [MJ/kg] |
| $Q_{ex}$ | heat energy released from solid propellant [MJ/kg] |
| $t_{ign}$ | igniter running time [s] |
| $\gamma$ | specific heat constant |
| $b$ | covolume [m$^3$/kg] |
| $r$ | radius of the grain [m] |
| $\dot{r}$ | burning rate [m/s] |
| $M$ | igniter charge mass [kg] |
| $C$ | propellant charge mass [kg] |
| $l_{ch}$ | chamber length [m] |
| $l_{ign}$ | igniter length [m] |
| $A$ | cross sectional area of chamber [m$^2$] |
| $n$ | pressure exponent |
| $a$ | coefficient of burning rate [m/s/pa] |
| $B$ | coefficient of burning rate |
| $D$ | outer diameter of grain [m] |
| $d$ | inner diameter of grain [m] |
| $L$ | grain length [m] |
| $\upsilon_p$ | volume of igniter [m$^3$] |
| $V$ | volume of chamber [m$^3$] |
| $c$ | sound speed in gas [m/s] |

Internal Ballistics (IB) model using two-step second order Lax–Wendroff method to capture the flame spreading in the propellant bed. The study shows that the improvement in numerical method is required to capture the abrupt change occurring in the flow variables during the combustion. Lowe [5] has considered a single phase flow model with an equation for reactant species and solved using a weighted average flux (WAF) method. The effect of propellant burning on pressure space curve is analyzed and the results are in good agreement with the experimental results. The Baer–Nunziato five equation model [6] has been studied for numerical analysis of gas – particle flows in the chamber using Rusanov scheme. The outcome is the maximum pressure, muzzle velocity and total time required for projectile to exit the muzzle and in acceptable range. It is suggested to improve the model to consider the combustion chamber with variable section. Florio [7] has developed a direct particle motion method and applied to simulate the phenomena occurring during burning of propellant grains. The parametric study shows that the system is highly sensitive to the pressure index. TWOPIB code [3] has been developed with fully implicit finite volume upwind difference scheme to study the influence of igniter and propellant charge position on internal ballistics. The experiments are also carried out for the same study and results are validated. The formation of shock wave can also be predicted with the same model. Mickovic et. al. [8] has studied two phase flow interior ballistics model using two analytical and one numerical method to obtain accurate pressure profiles behind the moving projectile. The study shows that the numerical method has great advantage over the analytical one. The effect of igniter vent holes and igniter length on formation of pressure wave inside the gun has been studied using Multi Phase Particle In Cell ($MP-PIC$) model [9] with Eulerian–Lagrangian approach. The outcome pressure space curve is in good agreement with the one obtained using IBHVG2. Recently, Jang et. al. [10] has made three different cases according to the propellant positions in the chamber, at breech, at center and at base of the projectile. The effect of these positions is observed in terms of the breech and base pressure and projectile acceleration where central position gives the best performance. However, the model was inadequate to study the pressure waves. Cheng et. al. [11] has studied interior ballistics two phase flow model with Computational Fluid Dynamics (CFD)-Discrete Element Method (DEM) approach with dynamics collision process of particles, which is mostly not considered in two phase reactive flow of interior ballistics model. The results can be used as an assessment tool for future studies and also for understanding the physical phenomena.

For simulation study of IB problem and its parametric analysis several numerical codes has been developed like TWOPIB [3], MGBC [1], CASBAR [12], XKTC & NGEN [13], PHOENICS [2] and FSPC [14].

In this work, focus is to study the spatial variation in pressure and its effect on the flow dynamics in the gun chamber. A numerical scheme has been identified and is used to study the motion of gases in the chamber. The numerical code has been generated by using FORCE scheme and simulated in MatLab. The solid propellant burning model, using gaseous ignition