Applied Mathematical Modelling 39 (2015) 2276–2294

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A phenomenological operator description of dynamics of crowds: Escape strategies

Fabio Bagarello [a], Francesco Gargano [b,*], Francesco Oliveri [c]

[a] DEIM – Facoltà di Ingegneria, Università di Palermo, Viale delle Scienze, I-90128 Palermo, Italy
[b] Dipartimento di Matematica ed Informatica, Università di Palermo, Via Archirafi 34, I-90128 Palermo, Italy
[c] Dipartimento di Matematica e Informatica, Università di Messina, Viale F. Stagno d'Alcontres 31, I-98166 Messina, Italy

ARTICLE INFO

Article history:
Received 1 November 2013
Received in revised form 4 July 2014
Accepted 30 October 2014
Available online 8 November 2014

Keywords:
Fermionic operators
Heisenberg-like dynamics
Dynamics of crowds
Escape strategies

ABSTRACT

We adopt an operatorial method, based on creation, annihilation and number operators, to describe one or two populations mutually interacting and moving in a two-dimensional region. In particular, we discuss how the two populations, contained in a certain two-dimensional region with a non-trivial topology, react when some alarm occurs. We consider the cases of both low and high densities of the populations, and discuss what is changing as the strength of the interaction increases. We also analyze what happens when the region has either a single exit or two ways out.

1. Introduction and preliminaries

In a recent paper, Ref. [1], two of us (F.B. and F.O.) used an operatorial approach, based on some well known features of quantum mechanical methods, to describe the interaction between two different populations constrained in a finite, closed, two-dimensional region. In particular, two different applications have been considered. In the first one, there are two populations located in different regions, a *poor area* and a different, *richer*, zone, and it is investigated how the two populations spread during the time evolution. In the second application, namely a simplified view to a predator–prey system, the two populations interact adopting essentially the same mechanisms as before, with the major difference that, at $t = 0$, they are located in the same area. Again, the main interest was in the time dispersion of the two species.

Here, adopting the same operatorial mechanism, we consider a different but somehow similar problem, for which a quite different interpretation is needed. We have two different populations, $\mathcal{P}_a$ and $\mathcal{P}_b$, forced to stay together in a certain two-dimensional region $\mathcal{R}$ with a non-trivial topology. What we have in mind is essentially the following: $\mathcal{P}_a$ and $\mathcal{P}_b$ are two groups of, say, young and aged people, staying in a shop (or in some closed space), with some exits and some obstacles around (like shelves, columns, ...). We want to explore what happens when some alarm starts to ring. How do the two populations react? How fast do they leave the room? How different the behaviors of the two populations are? Is there any reasonable way to help $\mathcal{P}_a$ and $\mathcal{P}_b$ to move faster? We will consider a somehow fixed topology, and a fixed initial condition, playing with the parameters of the model and with the escape strategies in order to find some sort of *optimal path*, or,

* Corresponding author.
E-mail addresses: fabio.bagarello@unipa.it (F. Bagarello), francesco.gargano@unipa.it (F. Gargano), francesco.oliveri@unime.it (F. Oliveri).

http://dx.doi.org/10.1016/j.apm.2014.10.038
0307-904X/

more generally, some optimal escape strategy. Also, we will consider the case in which the two populations do not interact, which is reasonable where there is only a small number of people inside the room at $t = 0$, and the case in which some interaction between $\mathcal{P}_a$ and $\mathcal{P}_b$ is expected, *i.e.*, for a sufficiently large number of people.

This kind of problem has clear implications in concrete situations, and for this reason it has attracted, along the years, the attention of several researchers who dealt with escape dynamics in many physical environments. Various methodological approaches to simulate crowd evacuation can be found in the vast scientific literature developed in the last years: each of these approaches has pros and cons, and clearly none of them can solve the problem completely. In fact, some approaches work better than the others according to the physical effects one wants to include in the model, to the level of the description (*microscopic* or *macroscopic*), or to the computational effort. Among these methods we can quote those based on lattice gas and cellular automata (see, for example, Refs. [2–7]) which revealed really consistent in several contexts: they are interesting microscopic models which often cannot properly simulate some typical *high-pressure* phenomena arising from the contact forces during the evacuation of a crowd. To account for these high-pressure phenomena, as well as other collective behaviors of a crowd, approaches based on force-models can be used (see, for example, Refs. [8–10]). The force-based description is generally motivated by the observation that the motion of pedestrians deviates in the presence of other pedestrians, and this effect seems as induced by a force that must be included in the model to correctly simulate the pedestrian motion. Nevertheless, some unrealistic behaviors may arise, especially for high densities: overlapping, oscillations phenomena, or backwards movement due to negative velocities. To avoid or to control these phenomena, the equations of motion must implement other procedures, *e.g.*, collision detection algorithms, having as a counterpart a substantial increasing of the complexity of the model. Methods based on the interactions of individuals or collective agents are taken into account in the agent-based-methods (see Refs. [11–13]): these methods rely on the techniques of cognitive science and they are able to capture or predict some *emergent* phenomena arising during a crowd evacuation, even if there are some difficulties to model a form of intelligence for each agent. The macroscopic point of view is generally the framework in which the fluid–dynamic models are built: the idea here is to consider the motion of a crowd like a fluid motion (see Ref. [14]). This viewpoint is quite reasonable, but it is limited by the requested hypothesis of high density of the crowd, which is not always the condition one wants to consider. Other recent methodological approaches are based on game theory (see Refs. [15,16]): even if these methods take into account some wanted effect of strategic thinking that can characterize, for obvious reasons, the behavior of a crowd, it is really difficult, especially with a large number of players, to find the appropriate payoff matrix required for a game. We refer the interested reader to some interesting review papers in which a more detailed analysis of the problem is performed (Refs. [17–19]).

Our approach to the problem of an escaping crowd is really different and it is based on operatorial methods of quantum mechanics explained in details in the next section. We only mention here that this kind of approach has revealed successful not only to describe the migration of a population, as previously said, but also the dynamics of several other macroscopic models, such as stock markets, love affairs or closed ecosystems (see Ref. [20–22]).

The paper is organized as follows. In Section 2, we introduce the Hamiltonian operator describing the dynamics of the populations $\mathcal{P}_a$ and $\mathcal{P}_b$, determine the equations of motion and write down the solution. We refer again to Ref. [20] for the general ideas behind our settings. In Section 3, we apply our strategy to the derivation of the dynamics of the two populations, and we discuss in details the meaning of the parameters of our model. Our conclusions are given in Section 4. Finally, Appendix A contains some snapshots of our video simulations which are available, for the interested readers, upon request to the authors.

2. The dynamical model

Let us consider a 2D-region $\mathcal{R}$ in which, in principle, the two populations $\mathcal{P}_a$ and $\mathcal{P}_b$ are distributed. The (*e.g.*, rectangular or square) region $\mathcal{R}$, is divided in N cells (see Fig. 1), labeled by $\alpha = (i,j),\ i = 1,\dots,L_x,\ j = 1,\dots,L_y$; to simplify the notation, when needed, we refer to the cell $(1,1)$ as cell 1, the cell $(2,1)$ as cell 2,..., the cell $(1,2)$ as the cell $L_x + 1$,...and the cell (L_x, L_y) as the cell N. In the rest of this paper, we will always assume that $L_x = L_y = L$. It should be stressed that, contrarily to what we have done in Ref. [1], here not all the cells can in principle be occupied, since there are obstacles in $\mathcal{R}$ where the populations cannot go. Moreover, $\mathcal{R}$ is not a closed region as it was in Ref. [1], but, on the contrary, there are exits somewhere on the borders, exits which $\mathcal{P}_a$ and $\mathcal{P}_b$ want to reach, as fast as they can, to leave $\mathcal{R}$ under some emergency.

As widely discussed in Ref. [20], in our approach the dynamics of the system $\mathcal{S}$ under analysis is defined by means of a self-adjoint Hamiltonian operator which contains all the mechanisms we expect could take place in $\mathcal{S}$. Following Ref. [1], we assume that in each cell α the two populations, whose related relevant operators are $a_\alpha,\ a_\alpha^\dagger$ and $\hat{n}_\alpha^{(a)} = a_\alpha^\dagger a_\alpha$ for what concerns $\mathcal{P}_a$, and $b_\alpha,\ b_\alpha^\dagger$ and $\hat{n}_\alpha^{(b)} = b_\alpha^\dagger b_\alpha$ for $\mathcal{P}_b$, are described by the Hamiltonian

$$H_\alpha = H_\alpha^0 + \lambda_\alpha H_\alpha^I, \quad H_\alpha^0 = \omega_\alpha^a a_\alpha^\dagger a_\alpha + \omega_\alpha^b b_\alpha^\dagger b_\alpha, \quad H_\alpha^I = a_\alpha^\dagger b_\alpha + b_\alpha^\dagger a_\alpha. \tag{2.1}$$

We refer to Ref. [1] for more details on the meaning of H_α. The operators involved in (2.1) satisfy the following anticommutation rules:

$$\{a_\alpha, a_\beta^\dagger\} = \{b_\alpha, b_\beta^\dagger\} = \delta_{\alpha,\beta}, \quad \{a_\alpha^\sharp, b_\beta^\sharp\} = 0. \tag{2.2}$$