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Towards a global controller design for guaranteed synchronization of switched chaotic systems

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ABSTRACT

In this paper, synchronization of identical switched chaotic systems is explored based on Lyapunov theory of guaranteed stability. Concepts from robust control principles and switched linear systems are merged together to derive a sufficient condition for synchronization of identical master–slave switched nonlinear chaotic systems and are expressed in the form of bilinear matrix inequalities (BMIs). The nonlinear controller design problem is then recast in the form of linear matrix inequalities (LMIs) to facilitate numerical computation by standard LMI solvers and is illustrated by appropriate examples.

1. Introduction

Synchronization of two chaotic systems was first introduced and experimentally demonstrated by Carroll and Pecora [1]. Since then chaos synchronization based schemes have found numerous applications in secure communication [2], cryptography [3], chaotic shift keying [4], chaotic modulation [5], etc. For a given chaotic system which is termed as the master or driving system, the concept of synchronization is to make the states of another identical chaotic system (termed as slave or response system) follow those of the master system. Many chaos synchronization methods have been investigated by contemporary researchers, like the linear feedback control [6], time delayed feedback control [7], artificial intelligence based optimization methods [8], nonlinear control [9], active control [10], adaptive sliding mode control [11], etc. In this paper, a nonlinear controller design technique is proposed in terms of BMI formulation for synchronization of switched chaotic systems, which are then recast as a set of LMIs. This makes the global nonlinear controller design procedure easy as standard numerical solvers [12,13] employing efficient interior point algorithms are available for solving such LMI problems in polynomial time.

Hybrid systems are ones in which both the continuous and discrete dynamics co-exist. Switched systems are a class of hybrid systems in which the discrete dynamics of the switching function is neglected [14]. A switching rule dictates the sequence in which the subsystems are switched and the time for which each subsystem is active. This switching rule can

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be arbitrary where the user has no control over it, or it may be controlled according to the designer's specifications. Hybrid systems are becoming more important due to their ability to model a large class of problems in biology [15], control systems [16], chemical reactions [17], communication networks [18], etc. Theoretical results regarding switched systems can be found in [14,19,20].

In this paper, we consider the case of synchronization of a unified chaotic system [21] where the master and slave systems simultaneously switch from one chaotic system to the other. The switching rule is not predefined and is assumed to be arbitrary. The objective is to design a single nonlinear controller so that the synchronization of the master and slave system is maintained in spite of the unknown arbitrary switching amongst various chaotic systems. Very few works have been done on switched chaotic systems [18], and the motivation of this paper is to develop a universal controller which synchronizes all members of the family of unified nonlinear chaotic systems even under arbitrary switching amongst them. Under such arbitrary switching among the unified chaotic system family, the synchronizing hardware i.e. the controller structure does not need to be changed every time. In many applications, the synchronizing control circuit can only be made once and should not be changed with the nature of the chaotic system. Designing dedicated synchronizing controllers for each different chaotic system may be infeasible to implement in many realistic scenario. Herein lies the applicability of designing a generalized global controller which can handle arbitrary switching among different chaotic system. From hardware implementation point of view, it is also economic to have a global controller. Such a controller, once designed can be treated as a plug and play device for synchronization between any chaotic systems amongst the unified chaotic system family.

There have been several efforts to study synchronization between similar and different chaotic systems e.g. using feedback control, time delay methods, fuzzy, impulsive control [22,23] and robust synchronization [24] for various applications like chaos based secure communication [25], etc. But most of the papers deal with a fixed structure of the chaotic system from which the error dynamical system can easily be obtained analytically. A stabilization scheme for the error dynamical system can then be used to ensure successful master–slave chaos synchronization. On the other hand, there have been huge advancements in the last few years on the application of robust stability analysis for switched linear systems and hybrid systems [14,26,27]. Lyapunov stabilization problem for a set of linear systems with arbitrary switching amongst them can be solved in terms of few set of LMIs which are derived from a successful bridging between the switched linear systems theory and robust stability theory. Unfortunately, robust stabilization problems for switching amongst complex nonlinear systems are not adequately addressed in literature and to the best of our knowledge, such results for switched nonlinear chaotic systems does not exist. In this paper, we formulate a synchronization scheme between two chaotic systems which continuously switches its characteristics within the Unified chaotic system family. Therefore, here we propose a new class of chaotic systems known as switched chaotic systems and also design a global control scheme to ensure guaranteed synchronization with the consideration of arbitrary switching in the master and slave chaotic system. Here, we study few interesting cases of switching phenomena in the chaotic system like switching of the key parameter (α) of the Unified chaotic system, along with the switching of the initial conditions, on–off switching of the synchronizing nonlinear controller, etc. It is shown that all of these cases can be synchronized with only one value of the controller gains. Hence this might be termed as a 'global' controller.

LMI problems are convex in nature and there are efficient interior point methods [28] to numerically solve such problems. Lyapunov stabilization scheme for switched systems often reduces to bilinear matrix inequality (BMIs) but these problems are generally non-convex in nature which cannot be solved directly using LMI solvers. In addition, the BMIs are mostly 'NP-hard' problems [29] where the class 'P' indicates problems solvable in polynomial time. Reducing the computational complexity to the solution of the NP hard problems is still an active research area. It is known that the NP-hard is not a characteristics for a particular algorithm but of the problem itself and in many cases such problems could be solved using some approximation or heuristics [30]. For example, the branch and bound algorithm [30] can be employed to solve BMIs but the objective function in such cases need to have a tight upper and lower bound. Also, it is well known that many problems encountered in control theory can be cast as BMI problems and there are many ways to solve BMIs. As in the case of LMIs, the computationally efficient interior point methods cannot be directly employed to solve BMIs. But depending on the structure of the control problem formulation, some mathematical simplification could yield a set of LMIs which can be easily solved. However, it is important to emphasize that such simplification of BMIs to be solvable using standard LMI tools using some analytical treatment or mathematical transformation is extremely problem dependent and as such cannot be considered as a generalized framework to solve all BMI problems. Otherwise BMI solution techniques use some sort of local search techniques which depends on the initial guess values and converges to a local minima. In such an approach there is no guarantee that the obtained solution is a global optima [31–33] e.g. the D–K iteration for μ-synthesis [34], alternating semi-definite programs (SDP) method [31], dual iteration method [35]. More details on the computational complexity of BMI problems arising in control and stabilization problems of switched systems and the use of coupled heuristic optimization and convex optimization or the memetic algorithms has been illustrated in Pan and Das [36]. However, in the present problem, the BMI formulation has been reduced to a set of LMIs with suitable transformation [37] so that the efficient interior point method based solution techniques can be applied. The robust control toolbox of Matlab [12] can be employed to solve the LMIs. The other popular software package YALMIP [13] gives more flexibility in implementing the LMIs and hence is widely used in the control community. Simulation and optimization based optimal controller design has been previously investigated in Das et al. [38] with a particular choice of initial conditions for establishing synchronization amongst two chaotic systems or the suppression of chaotic oscillation in highly complex systems [39], without the consideration of switching. But such synchronization is not theoretically guaranteed for all possible initial conditions. This is addressed in the present paper with the Lyapunov stabilization and a BMI formulation for the switched chaotic system scenario.