Applied Mathematical Modelling 39 (2015) 2363–2384

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Second-order nonlinear dynamics of catenary pipelines subjected to bi-chromatic excitations

Ioannis K. Chatjigeorgiou *

School of Naval Architecture and Marine Engineering, National Technical University of Athens, 9 Heroön Polytechniou Ave, GR157-73, Zografos Campus, Athens, Greece

ARTICLE INFO

Article history:
Received 3 August 2013
Received in revised form 3 June 2014
Accepted 10 November 2014
Available online 20 November 2014

Keywords:
Pipelines
Nonlinear dynamics
Sum-frequency
Difference-frequency
Perturbations

ABSTRACT

The purpose of this study is to investigate the 2D nonlinear dynamics of catenary pipelines for marine applications approaching the solution in the frequency domain. The proposed methodology is seeking the transfer functions and in particular the quadratic magnification factors of the involved dynamic components assuming a bi-chromatic excitation, namely, a signal which arises from the superposition of two different sinusoidal harmonics. The sought solution is achieved by employing a perturbation technique that expands all dynamic components into series of perturbations relatively to a scaling factor, whilst the mathematical processing is performed into the complex space.

The adopted procedure results in a series of problems which are solved separately and successively. Also, separate mathematical systems are derived for the sum- and the difference-frequency problems. The numerical results are obtained using a centered-differences approximation of the final set of ordinary differential equations. The correlation of the structural model with marine applications required the employment of a proper procedure for linearizing the nonlinear drag force at second-order. Finally, it is remarked that the outlined methodology can be effectively extended to polychromatic excitations and to the 3D space as well.

1. Introduction

The present study aims at approaching the nonlinear dynamic behavior of catenary pipelines conveying fluids using a frequency domain based technique. The main task is to include the effects of bi-chromatic excitations, namely, the combination of two regular harmonics that induce the occurrence of sum- and difference-frequency effects. The outlined theory accounts for both counterparts, whilst only the former are considered in the computations for reasons explained adequately in the dedicated section.

The study of nonlinear dynamic problems associated with constantly vibrating continuous structural systems (such as the present structural model) relies mainly on the employment of time domain techniques, which are able to tackle any kind of nonlinearities and any type of excitations, from simple sinusoidal to random signals. Time domain techniques do not concern about nonlinear terms and their physics; they just embrace them, without being able, however, to explain the details of their contribution. The outputs are typically time histories of the involved dynamic components, whilst a further analysis on the details of the response requires laborious post-processing of the results. The approach of the sought solution in the time

* Tel.: +30 210 772 1105; fax: +30 210 772 1412.
E-mail address: chatzi@naval.ntua.gr

http://dx.doi.org/10.1016/j.apm.2014.11.010
0307-904X/

domain dictates the employment of purely numerical methods (with few exceptions of simplified nonlinear mathematical systems). However, although the time domain solution methods can provide effectively the output signals of the response, the thorough interpretation of the occurrence of nonlinearities require further post processing of the results, e.g. using Fast Fourier Transformation techniques.

To provide further insight on the contribution of the nonlinear terms in the case of the nonlinear dynamics of marine riser-type structures, Chatjigeorgiou [1] adopted a frequency domain approach and derived the quadratic transfer functions of the governing dynamic components considering monochromatic excitations. In the same study, it was shown that the mechanism which stimulates the second-order double-frequency nonlinear contributions can be regarded as a distribution of generalized springs that originate from products of the linear counterparts.

In practical applications, however, and especially in the environment where marine risers typically operate, the assumption of monochromatic (a single harmonic) excitation is indeed simplistic. The motion of the host facility, which accordingly drives the top end (the connection point) of the pipeline, follows the elevation of the free surface which can be approximated by the superposition of multiple harmonics. Hence, the riser-type structures are excited by polychromatic motions, the simplest approximation of which, are the bi-chromatic forced displacements. Although the present study tackles explicitly the problem of two harmonics, it should be mentioned that the theory and the methodology outlined in the main text can be further extended to incorporate additional harmonics.

The employment of the frequency domain to elaborate a nonlinear problem was inspired by analogous studies in hydrodynamics, where the nonlinear free surface boundary condition in bi-chromatic propagating waves effectively describes a pressure distribution that induces sum- and difference-frequency terms. Indicative studies on the subject are those due to Kim and Yue [2], Eatock Taylor and Huang [3] and Choi et al. [4]. Nevertheless, in hydrodynamics, the perturbation technique is applied primarily on the nonlinear free surface boundary condition and secondarily on the field equation, namely, the Laplace equation. In the present structural dynamics problem, the perturbation technique is applied directly on the mathematical system describing the dynamic equilibrium of the pipeline.

Admittedly, there are alternative and efficient methods that treat nonlinear problems in the frequency domain, such as the harmonic balance [5], the incremental harmonic balance [6] and the shooting methods [7]. Those, however, do not cancel the value of the proposed methodology, which could be effectively applied for a variety of nonlinear structural dynamics problems as well.

The text flow of the present contribution is organized as follows: Section 2 provides the governing nonlinear dynamic set of equations; Section 3 introduces the perturbation technique, whilst the final relations, together with the drag force linearization for bi-chromatic excitations, are provided in Section 4; relevant numerical results are given in Section 5 followed by discussion.

2. The two dimensional nonlinear dynamic structural system

The catenary pipeline is modeled as an Euler–Bernoulli beam having a curved line configuration in its 2D plane of reference (Fig. 1). To resemble the function of a riser, the catenary pipeline is considered simply supported with both ends pinned having a perpendicular spacing, equal to the depth of installation. The catenary configuration is attained by imposing a specific pretension force at the top end or by specifying the horizontal spacing between the two pinned ends.

Let m, m_a, w, d_O, d_I, A and I denote respectively, the mass, the hydrodynamic mass, the submerged weight (defined per unit unstretched length), the outer diameter, the inner diameter, the cross sectional area and the related second moment of the pipe. All components correspond to the unstretched condition. The density of the pipe is denoted by ρ_s and the Young's modulus of elasticity by E. Accordingly, the axial rigidity and the bending stiffness will be given by EA and EI respectively. It is assumed that the pipeline conveys a fluid with steady velocity V. The density of the fluid is denoted by ρ_f and yields a fluid mass per unit unstretched length of the pipe equal to M. The inner flow is described by the so-called "plug flow" model [8], i.e. it is assumed that fluid velocity inside the pipe maintains a constant profile along its complete length. Torsion effects are neglected and the assumption is made that the stress–strain relation is linear whereas the elastic deformation along the pipe is nearly zero ($e \approx 0$). A Lagrangian coordinate system is introduced which is defined by the unit vectors **t** and **n**, where **t** is

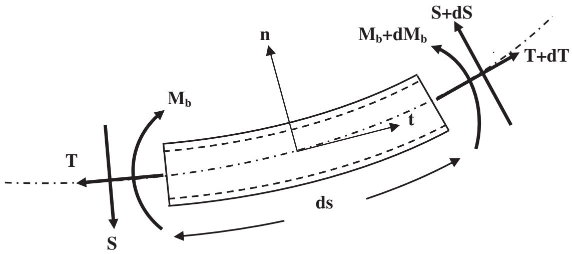

Fig. 1. Internal loading components acting on an element of the curved beam: **T**, **S** and **M**, denote respectively the tension, the shear force and the bending moment, all defined in the 2D plane of reference.