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Power law fluid flow through a bundle of regular fibers

J.A. Kolodziej [a], M. Mierzwiczak [a,*], M. Ciałkowski [b]

[a] Poznan University of Technology, Institute of Applied Mechanics, ul. Jana Pawla II 24, 60-965 Poznan, Poland
[b] Poznan University of Technology, Institute of Thermal Engineering, ul. Piotrowo 3, 60-965 Poznan, Poland

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ABSTRACT

The numerical predictions of the velocity field and the component of longitudinal permeability in the filtration equation for the steady incompressible flow of power law liquids through the assemblages of cylindrical fibers are presented in this paper. The fibers are arranged regularly in triangular, square, and hexagonal arrays. Flow is longitudinal with respect to the fibers. The non-linear governing equation in the repeated element of the array is solved using Picard iteration. At each iteration step the method of fundamental solutions and the method of particular solutions are used. The advantage of the proposed meshless approach is that it does not require the generation of a mesh in the domain or its boundary, but uses only a cloud of arbitrarily located nodes. The non-homogenous term is interpolated by radial basis functions. The next bundle of fibers is treated as a porous media and on the base of velocity field the permeability coefficients are calculated as functions of porosity.

Contents

1. Introduction

Fluid flow through a fibrous porous media plays an important role in various engineering situations. Common examples include the resin transfer molding process, industrial filters, insulating materials, and flow in spinning artificial fibers. The knowledge of permeability which characterizes the ability of fluid to penetrate the fibers is an important factor in the design of the above mentioned processes and devices.

* Corresponding author. Tel.: +48 61 665 2387; fax: +48 61 665 2307.
E-mail addresses: magdalena.mierzwiczak@put.poznan.pl, magdalena.mierzwiczak@wp.pl (M. Mierzwiczak).

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In the case of parallel arrangement of fibers due to the anisotropic properties of the porous media, the laminar flow through such media is modeled by Darcy's law in the form:

$$\mathbf{q} = -\frac{\mathbf{K}}{\mu}\nabla p, \tag{1}$$

where $\mathbf{q}$ is filtration velocity, p denotes the fluid pressure, μ is the fluid viscosity, and $\mathbf{K}$ is the permeability tensor.

For the fibrous media with unidirectional fiber arrangement the permeability tensor in a coordinate system with the z-axis parallel to the fibers axis has the form

$$[K] = \begin{bmatrix} K_{\perp x} & 0 & 0 \\ 0 & K_{\perp y} & 0 \\ 0 & 0 & K_{II} \end{bmatrix}, \tag{2}$$

where $K_{\perp x}$ and $K_{\perp y}$ are the components of the permeability tensor in the principal directions perpendicular to the fibers, and K_{II} is the component of the permeability tensor in the direction parallel to the fibers.

For a Newtonian fluid the components of the permeability tensor are dependent only on the porosity and the geometric structure of the fibers. There have been many analytical, numerical, and experimental papers in which these components were determined for a Newtonian fluid. We briefly discuss papers in which the authors assume a system with a regular structure of fibers because in our considerations such a structure is assumed.

The authors usually consider flow in a repeated element of regular array of fibers, as micro structural flow problems, for the determination of the permeability components.

The various authors have considered both longitudinal flow with respect to the fibers – the determination of K_{II} [1–10] and transverse flow with respect to the fibers – the determination of $K_{\perp x}$ and $K_{\perp y}$ [3,5,11–18]. In the first theoretical papers, the cell models were proposed by Happel [5] and Kuwabara [14]. These models assume that the fibers are spaced apart far enough to divide the medium into independent cells. Typically one uses a circular cell, with the fiber located in the center with appropriate boundary conditions on the outer boundary of the cell. The size of the cell is a function of porosity (manner of fiber arrangement). Hasimoto [11] used a Fourier series method to solve the Stokes equations for transverse flow in a square array of fibers but his method is only valid for very high porosities. For low porosities, when the fibers are closely spaced, a lubrication approximation was used in [12] for the flow-rate pressure drop relationship and an analytical expression for the transverse permeability was obtained. Boundary collocation methods have often been applied to solve micro structural flow problems in a repeated element of a regular array [19]. Such methods were used for the longitudinal flow in a triangular array in [2,9,10] while for a square array in [2,7,9], for a hexagonal array in [6] and for a rectangular array in [2]. For transverse flow in a triangular and square array such a method was used in [17].

Usually the authors consider the problem of determining the permeability components assuming an infinite array of fibers. The problem of an array with a boundary was considered in [7,8,15]. Such a consideration permits determine of the additional macroscopic characteristics of fibrous porous media such as the effective viscosity or the sleep constant in the Beavers–Joseph boundary condition.

Large discrepancies are frequently observed between theoretical results and experimental data obtained in real fiber beds for $K_{\perp x}$ and $K_{\perp y}$. The experimental results give a large value of permeability (smaller resistance to flow) in comparison with values obtained for almost all theoretical models with a regular arrangement of fibers. This discrepancy may be justified by the fact that in an experimental examination, especially at a low volume fraction of fibers, it is very difficult to maintain a homogeneous distribution of fibers which is assumed in a theoretical or a computational investigation. In fact, for real bundles of fibers, the distribution of fibers is more or less heterogeneous and regions with smaller packing density of fibers can considerably reduce the flow resistance. In recent years, several authors have investigated, either analytically or numerically, the effects of fiber size variation, perturbed fiber positions, and fiber lattice imperfections on the transverse permeability of unidirectional fiber arrays [20–22,19,23–25]. In many important practical situations fibers are not unidirectional but they create more complicated structures. Then in recent years, papers in which permeability in such structures are determined have appeared [26–29].

Another assumption that is usually made while determining the permeability of the medium is the approximation of creeping flow (Stoke's approximation). Only papers [30,16,31,13] have considered transverse flow with respect to a regular unidirectional array of fibers. All these authors use the Navier–Stokes equations (finite Reynolds numbers). In such a case, the permeability is a function of not only the porosity and structure but also of the Reynolds number, so we are dealing with the non-linear filtration equation.

All the papers mentioned above consider Newtonian fluid flow in a fibrous porous medium. Papers in which non-Newtonian fluid flow through fibrous media is considered are very few. The Stokes equation in a square and a hexagonal arrangement of fibers for mildly shear-thinning fluids for transverse flow only was considered in [32]. Vijaysri et al. [33] studied the perpendicular flow of a power-law fluid across an array of cylinders using a cell model and the finite difference method (FDM). A higher order Galerkin finite element method (FEM) was used in [34] for the flow of viscoelastic fluids past periodic square arrays of cylinders. The authors consider inertial, shear thinning viscosity and elasticity effects. In [35] the finite volume method (FVM) was used for the calculation of the flow of viscoplastic materials through tube bundles.