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A modified tunneling function method for non-smooth global optimization and its application in artificial neural network

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A B S T R A C T

For solving a class of non-smooth unconstrained global optimization problems, we present a novel definition of the modified tunneling function which combines the characters of tunneling function and filled function, and then give a one-parameter modified tunneling function. Issues covered in the presented work include: theoretical properties, solution algorithms and numerical experiments. Furthermore, an improved artificial neural network hydrological forecasting method using the modified tunneling function is also reported. The preliminary experiment results confirm that the proposed approach is promising.

1. Introduction

This paper considers a class of non-smooth unconstrained global optimization problems as follows:

$$\min_{x\in R^n} f(x).$$

We formulate the following three hypotheses.

H1. $f : R^n \to R$ is Lipschitz continuous;
H2. The number of minimum points of $f(x)$ can be infinite, yet the number of different function values at the minimum points is finite; and
H3. $f(x) \to +\infty$, as $\|x\| \to +\infty$.

Analytically, these assumptions imply that there exists a closed bounded domain $\Omega \subset R^n$ such that Ω contains all minimum points of $f(x)$ and the function value when x is on the boundary of Ω is greater than any function value when x is inside Ω. In this case, we assume that Ω is known and our approach only considers the points in Ω. That is, we consider the following equivalent problem (P):

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$$(P) \quad \min_{x\in\Omega} f(x).$$

Assume that x_1^* is a current local minimum point of $f(x)$.

Global optimization, as a powerful solution method, finds wide applications in many fields such as finance, engineering, management and social science. The tunneling function method, first proposed by Levy and Montalvo used for the unconstrained optimization[1], and recently reconsidered by Wei, Cetin and yang which extended to constrained optimization[2–4], is one of the effective deterministic global optimization methods. It is advanced in optimizing rate and the effect of optimization when compared with the stochastic one. It modifies the objective function as a tunneling function, and then finds a better local minimum point gradually by optimizing the tunneling function constructed on the minimum point previously found. The tunneling function method is composed of a sequence of cycles, which each cycle has two phase: a local minimization phased and a tunneling phase. From this point, we find that the tunneling function method is similar to that for the filled function method which is also an effective deterministic global optimization method[5]. In the open literatures, there are many papers devoted to the filled function method; for example, see [6–10].

The tunneling function method is originally introduced to solve smooth global optimization problem. However, there are several shortcomings in these tunneling functions. The main drawback of them is that it is need find another local minimum point x_1 and build $f(x_1) > f(x_1^*)$, which results in the necessity of adding another pole to the second local minimum point x_1, thus restarting the algorithm and calling for more workload. Later, Yang in [4] revised the definition of tunneling function and proposed a new tunneling function. The tunneling function method has been further investigated in [2–4], and more tunneling functions were proposed. It should be noted that the foregoing mentioned tunneling function methods are all proposed for solving smooth global optimization problems. However, in real world, many problems are allowed to be modeled as non-smooth global optimization problems. Some researchers [7,8,11] have extended the idea of filled function method to non-smooth global optimization. To address such situations, in this paper, drawing inspiration from the papers [7,8], we modify the definition of the tunneling function introduced in [1] and propose a modified tunneling function for non-smooth global optimization without special structure. The modified tunneling function contains only one parameter and it can be easily adjusted. Moreover, the modified tunneling function method can also be applied to smooth global optimization. In general, there are two issues faced by global optimization: one issue is how to find a better minimum point of the problem (P) starting from the current local minimum point; the other is how to verify the current minimizer is a global one. Our paper mainly aims to tackle the former issue.

The market for hydrological forecasting using artificial neural network technologies is experiencing strong growth. The artificial neural network that is trained by BP algorithm provided a more flexible solution based on function approximation. However, at present, the conventional BP approach is liable to trap in local optima and is slow rate of convergence. Taking it into consideration and trying to make a difference, this paper proposes an improved artificial neural network approach for hydrological forecasting, which is based on modified tunneling function method and aims at global optimization. The approach integrates modified tunneling function method into artificial neural network, thus results in a better situation, in which the local minimum point found by BP algorithm will be escaped and a latter minimum point, whose value of function is less than the previous one, can be found. This situation will repeat itself in circles until a global minimum point emerges. It has been proved by research that this experimental approach is able to improve the forecasting accuracy and that it shows great applicability.

The remainder of the paper is organized as follows. In Section 2, we present a modified tunneling function and investigate its properties, then we state the modified tunneling function Algorithm MTFM and perform a numerical experimentation. In Section 3, the artificial neural network hydrological forecasting Algorithm T-BP based on the modified tunneling function method is suggested and example results are given and analyzed. Finally, conclusions and some possible further studies are mentioned in Section 4.

For ease of explanation, let L be the Lipschitz constant of $f(x)$ with $\partial f(x)$ be the generalized gradient of $f(x)$ at the point $x, L(P)$ be the set of local minimum points for problem (P) with $G(P)$ be the set of the global minimum points for problem (P).

2. The modified tunneling function method for non-smooth global optimization

2.1. The background of filled functions and tunneling functions

The filled function method, initially introduced by Ge and Qin [11], and lately reconsidered by such as Zhang, Xu and Sahiner [6–10], is an effective deterministic global optimization method. The filled function method is composed of a sequence of two-phase cycles, local minimization and filling. In the first phase, we start from a given initial point and use any local minimization algorithm to find a local minimum point x_1^* of $f(x)$; In the second phase, we construct a filled function at x_1^* and minimize the filled function in order to find a point $\bar{x}$ with $f(\bar{x}) < f(x_1^*)$. Thus, we can use $\bar{x}$ as a new initial point in the first phase again, and then find a better local minimum point x_2^* of $f(x)$ with $f(x_2^*) < f(x_1^*)$. This process loops until a stopping criterion is met. The last local minimum point will be then taken as a global minimizer of $f(x)$.

Most of existing filled functions focus only on solving smooth global optimization problems, thus some researchers[7,8,11] have generalized the applicable area of the filled function method to non-smooth global optimization. We present the definition of filled function in non-smooth global optimization from [7] as follows.