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Notes on the exponentiated half logistic distribution

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ABSTRACT

In this paper moment estimators and maximum likelihood estimators of unknown parameters in the exponentiated half-logistic distribution are derived, and an entropy estimator is obtained for the distribution. An exact expression of Fisher information is derived to obtain approximate confidence intervals for unknown parameters in the distribution, and for illustration purposes, the validity of the proposed estimation method is assessed using real data.

1. Introduction

Gupta et al. [1] proposed the concept of an exponentiated distribution and discussed an exponentiated exponential distribution (EED) with two parameters (its scale and shape). Gupta and Kundu [2] considered the EED as a alternative to the gamma distribution (GD) and the Weibull distribution. Mudholkar and Srivastava [3] introduced an exponentiated Weibull distribution for modeling the bathtub failure rate based on lifetime data. For climate modeling, Nadarajah [4] proposed a generalization of the Gumbel distribution referred to as the exponentiated Gumbel distribution and provided its mathematical properties. Recently, Kang and Seo [5] discussed the estimation of the scale parameter and reliability function of the exponentiated half-logistic distribution (EHLD) based on progressively Type-II censored samples.

The cumulative distribution function (cdf) and probability density function (pdf) of the random variable X with the EHLD can be given by

$$F(x) = \left(\frac{1 - e^{-\theta x}}{1 + e^{-\theta x}}\right)^{\lambda},$$

and

$$f(x) = \theta\lambda\left(\frac{1 - e^{-\theta x}}{1 + e^{-\theta x}}\right)^{\lambda} \frac{2e^{-\theta x}}{1 - e^{-2\theta x}}, \quad x > 0,\ \theta, \lambda > 0, \tag{1}$$

where θ is the reciprocal of the scale parameter and λ is the shape parameter.

Fig. 1 presents the pdf of the standard EHLD for various shape parameters. It is observed that the pdf of the standard EHLD is a decreasing function for $\lambda \leqslant 1$, whereas it is a right-skewed unimodal function for $\lambda > 1$. These properties resemble those of the GD and the EED. As a special case, if $\lambda = 1$, then the EHLD is the half-logistic distribution.

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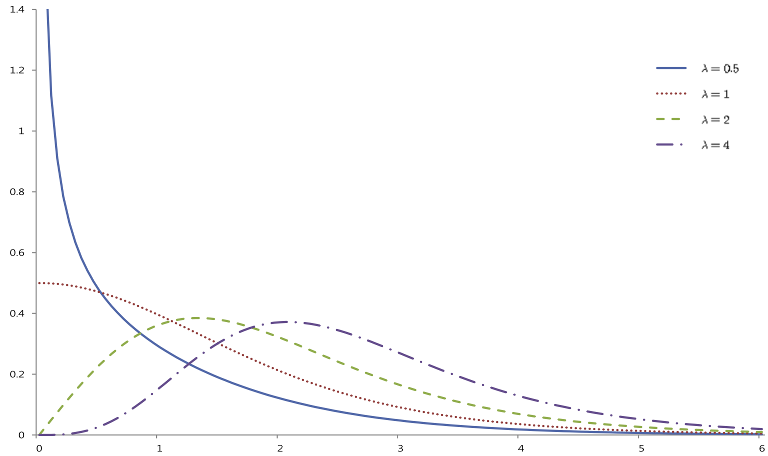

Fig. 1. The pdf of the standard EHLD for various shape parameters.

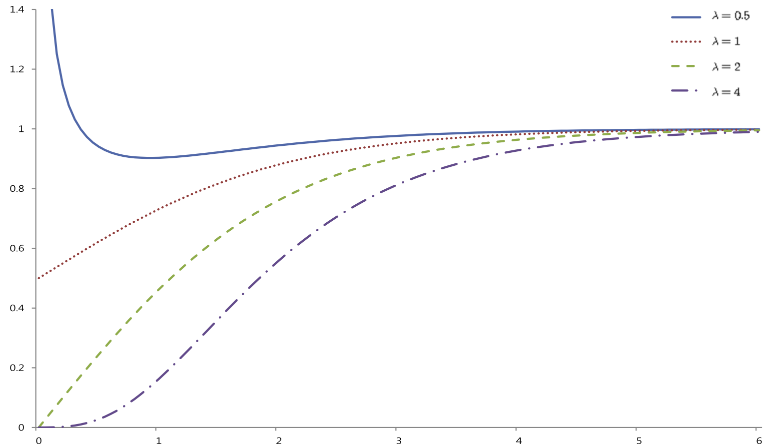

Fig. 2. The failure rate function of the standard EHLD for various shape parameters.

The failure rate function of the EHLD can be expressed as

$$h(x) = \frac{2\theta\lambda e^{-\theta x} F(x)}{(1 - e^{-2\theta x})(1 - F(x))}.$$

Note that the failure rate function approaches θ as $x \to \infty$ based on L'Hospital's rule. The failure rate function of the standard EHLD for various shape parameters is plotted in Fig. 2. For $\lambda < 1$, it decreases to a positive constant and then increases to θ. For $\lambda \geqslant 1$, it increases to θ, which is similar to the behavior of the failure rate function of the GD and the EED, whose shape parameters exceed 1. Gupta and Kundu [6] examined the behavior of failure rate functions of the GD and the EED. On the other hand, because the cdf of the GD does not have a not closed form unless the shape parameter is an integer, its reliability and failure rate functions cannot be expressed as a closed form. However, the cdf of the EHLD has an explicit expression.

Section 2 develops some theorems for relationships between the EHLD and well-known probability distributions. Section 3 derives the method of moments estimators (MMEs), maximum likelihood estimators (MLEs), and an entropy estimator for the EHLD. Section 4 discusses Fisher information and approximate confidence intervals (CIs) for unknown parameters in the EHLD. Section 5 uses a real data set to check whether the EHLD fits the data better than other well-known distributions and assesses the validity of the proposed method, and Section 6 concludes the paper.

2. Related distributions

Suppose that X is a random variable with the exponentiated half-logistic pdf given in (1). Then some known distributions related to the EHLD can be derived using Theorem 2.1.5 in [7, p. 51].