Applied Mathematical Modelling 39 (2015) 6517–6528

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modeling wave propagation in annular sector plates using spectral strip method

Saeed Bahrami [a], Fatemeh Shirmohammadi [b,*], Mohammad Mehdi Saadatpour [a]

[a] Department of Civil Engineering, Isfahan University of Technology, Isfahan, Iran
[b] Department of Civil Engineering, Kansas State University, Manhattan, KS, USA

ARTICLE INFO

Article history:
Received 17 July 2013
Received in revised form 26 January 2015
Accepted 5 February 2015
Available online 24 February 2015

Keywords:
Annular
Wave propagation
Dynamic stiffness
Natural frequency
Dynamic

ABSTRACT

A spectral strip method (SSM) in frequency domain was developed to analyze annular Levy-type plates subjected to impact loads. The dynamic formulation was developed for an annular sector plate with simple boundary conditions on radial edges and any boundary conditions on circular edges with constant and variable thickness. Using Bessel's equations, modified Bessel's equations, and separation of variation technique, the dynamic differential equation of annular sector plate in frequency domain is solved. Numerical results show the accuracy of this method in comparison to other numerical methods using less number of elements (strip).

Published by Elsevier Inc.

1. Introduction

Because dynamic characteristics of structures are essential in engineering, many diverse structural analysis methods, such as Finite Element Method (FEM), Finite Difference Method (FDM), and Boundary Element Method (BFM), have been developed over the last few decades to predict these characteristics. FEM, the most practical method in this field, provides an almost accurate dynamic characteristics of structural systems if the mesh size of the system under consideration is fine compared to the lowest wavelength. The vibrating shape of a structure changes with vibration frequency and FEM typically uses frequency-independent shape functions; therefore, FEM requires fine mesh to obtain solution with reasonable accuracy, especially in high frequencies. However, refining the mesh (h-method) or increasing the degree of polynomial shape functions (p-method) dramatically increases the cost of solving problems and discretization errors also increase remarkably. Using frequency-dependent shape functions is an alternative solution resulting in a stiffness matrix referred to as the dynamic stiffness matrix in the literature. Dynamic stiffness matrix, formulated by solving the governing differential equation, includes all features of the dynamic system, such as mass and stiffness. Since, to calculate the dynamic stiffness of a structural member, subdivision of structural member into finite elements is not required, so the discretization errors may be decreased and computational cost is decreased.

The structural analysis method which uses dynamic stiffness matrix known as Spectral Element Method (SEM) in the literature. SEM is similar to FEM in some aspects; structural discretization and assembling procedure are utilized in both methods. However, they have two major differences. First, in SEM the dynamic stiffness matrix is formulated in the frequency domain by frequency-dependent shape functions, achieved by an exact solution of governing differential equations

* Corresponding author.
E-mail addresses: s_bahrami@cv.iut.ac.ir (S. Bahrami), fatemeh@ksu.edu (F. Shirmohammadi), mmehdi@cc.iut.ac.ir (M.M. Saadatpour).

http://dx.doi.org/10.1016/j.apm.2015.02.007
0307-904X/Published by Elsevier Inc.

in the frequency domain. Second, SEM uses Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) algorithms to transform dynamic loads in time domain into frequency domain and construct response time history from frequency history [1].

Although, many researches have been conducted about dynamic stiffness matrix and using FFT and IFFT to calculate the dynamic characteristics of a system, according to our knowledge, Doyle is one of the first researchers applied SEM to wave propagation in structures. In 1986, he began working on wave propagation by elucidating a number of problems arising from the FFT algorithm, considering the impact load and the resulting wave propagation [2]. He and his colleagues have significantly impacted progress of SEM for diverse structural elements such as rode [3], beam [4], layered solids [5], multiple connected Timoshenko beam [6,7], varying cross-section [8], and spectral super element [9]. Their work in this field was based on research at Purdue University and is summarized in a book published in 1997 [10].

In addition to Doyle, Usik Lee and his colleagues have significantly contributed to the development of SEM. Before 1996, all developed structural elements were subjected to concentrated applied loads. Lee and Lee developed an original extended SEM to analyze a beam under dynamic distributed loads [1,11]. One of the Lee's recent articles discusses wave propagation in an extended Timoshenko beam [12]. A summary of Lee's work is presented in a book published in 2009 [13].

In 2011, Shao and Wu [14] studies the problem of thin plates resting on Winkler foundations with irregular domains by combining the Fourier spectral method with the differential quadratic method. In their work, distributed loading was approximated by Chebyshev polynomials. In addition, Sun and Lou [15] presented wave propagation and transient response of an infinite functionally graded plate under a point load. In their work, the integral transform was used to obtain analytical relation of the dispersion. A complete discussion of dispersion on functionally graded plates is provided in their paper.

In 2012, Park and Lee used SEM to formulate composite beams in the frequency domain. In their paper, axial-bending coupled equations of motion and boundary conditions were derived for two-layer smart composite beams using Hamilton's principle with Lagrange multipliers [16].

In all aforesaid researches, the developed structural elements were subjected to only the impact loads. Shirmohammadi et al. developed SEM to analyze moderately rectangular thick plate under moving load in addition to impact loads [17–19].

The purposes of this study include development of the spectral dynamic stiffness matrix for annular sector Levy-type thin plate, utilization of the derived dynamic stiffness matrix to extract natural frequencies and displacements of the plate with uniform and tapered thickness subjected to impact loads, and demonstration of the efficiency of the developed method compared to other numerical methods using the least number of elements to solve a desired problem, acknowledging it may not affect the computational time.

2. Equation of motion and spectral element modeling

Consider an annular sector thin plate (Fig. 1) subjected to external dynamic load $f(r,\theta,t)$. Assuming small displacements theory, the equation of motion under external applied load with damping property is given as [20]:

$$D\nabla^2\nabla^2 w + \rho h \frac{\partial^2 w}{\partial t^2} + \eta h \frac{\partial w}{\partial t} = f(r,\theta,t), \tag{1}$$

where

$$\nabla^2\nabla^2 w = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right), \tag{2}$$

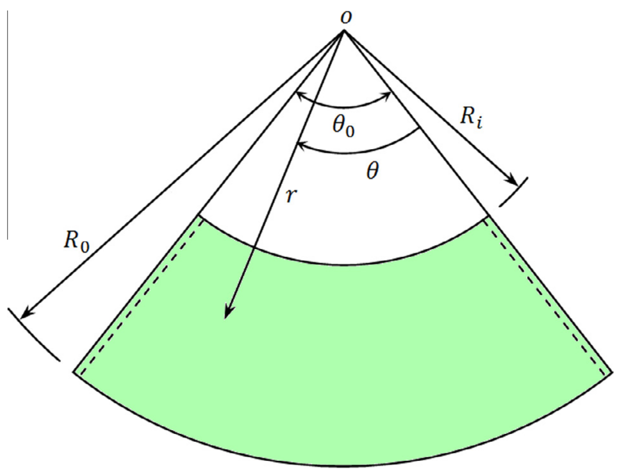

Fig. 1. The annular sector plate.