Applied Mathematical Modelling 39 (2015) 6580–6598

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

On coupled heat transport and water flow in partially frozen variably saturated porous media

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ARTICLE INFO

Article history:
Received 17 January 2014
Received in revised form 1 February 2015
Accepted 5 February 2015
Available online 26 February 2015

Keywords:
Frost action
Hygro-thermal model
Porous media
Rothe method
Pseudo-monotone operators
Finite element discretization

ABSTRACT

We propose and analyze a model of coupled heat transport and water flow in variably saturated porous media under conditions of freezing and thawing. Solution of the model provides temperature, liquid water content and ice content in porous media as a function of space and time. We employ the Rothe method to construct systems of approximate solutions for which the existence and regularity are proved. Numerical scheme is based on specific semi-implicit discretization in time, which preserve the pseudo-monotone structure of the discrete problem. Numerical results are compared with experimental data reported in the literature. The model satisfactorily predicts characteristic phenomena in porous media under frost actions such as water movement toward freezing fronts and the presence of liquid water in frozen regions.

1. Introduction

Coupled transport processes involving partially frozen porous media or fractured rock masses become important in civil engineering, soil science, agriculture industry or environmental engineering, in particular, in the construction of roads, railroads or tunnels, investigation of deterioration of geomaterials in frost regions, as well as hydrological and climate simulations. Freezing processes were already being explored and described in the last decades and several specific phenomena have been reported in literature [1]. One of the most important observations was that the water in pores does not freeze at 0 °C, further, in frozen porous media, not only ice, but also thin films of unfrozen water exist on the surface of porous particles. Consequently, there is not sharp interface between frozen and unfrozen regions in the porous media. Freezing processes affect water fluxes in frozen regions, cause significant changes in hydraulic conductivity and induce temperature gradients driving water movements toward freezing fronts.

Several more or less complex physical models describing coupled heat transport and water flow in partially frozen porous media and related phenomena have been developed. All hygro-thermal transport models build on a system of conservation laws of mass and energy including the phase transition of water (see e.g. [2–7]). Under frost action, the fundamental issue regarding the relationship between pressure head, water content and temperature must be solved. In this work, assuming thermodynamic equilibrium conditions we use the generalized Clapeyron equation to relate the pressure head with the temperature to derive the freezing characteristic curve. This thermodynamic relation is incorporated into balance equations in an appropriate way to cover the freeze–thaw process in porous media.

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http://dx.doi.org/10.1016/j.apm.2015.02.011
0307-904X/

Significant effort has been invested in the mathematical analysis of transport models for describing the response of wet porous materials to aggressive chemical reactions. For theoretical analysis of coupled reaction-infiltration models describing isothermal geochemical processes we refer to [8,9]. For related models and mathematical analysis in the context of concrete carbonation see e.g. [10–13]. Our contribution in this direction is more theoretical analysis of hygro-thermal responses of porous media subjected to frost actions. Although physical and computational models of coupled transport processes in porous media with phase transitions have been investigated by means of experimental and numerical approaches by several authors (see [5–7] and the references therein) deeper mathematical analysis of such models remains so far largely unexplored. Dalík et al. [14] analyzed the discrete-time solution of the one-dimensional Kiessl model [15] for moisture and heat transfer in ceramic and building materials. The existence of a local-in-time strong solution for moisture and heat transfer in multi-layer porous structures modeling by the doubly nonlinear parabolic system is proven in [16]. Vala in [17] proved the existence of the solution to the purely diffusive hygro-thermal model. In [18], the authors proved some existence and regularity of approximate solutions for the two-dimensional Bažant-Thonguthai model describing hygro-thermal behavior of concrete at high temperatures. In the present paper, we extend the analysis mentioned above to N-dimensional problems ($N = 1, 2, 3$) of coupled transport processes in porous media under frost action. Moreover, contrary to results mentioned above, we consider only positivity of transport coefficients, which is essential e.g. for hydraulic conductivity of soils. This yields degeneracies in both mass and energy balance equations. Further difficulty lies in the convective term in the heat equation, which represents strong nonlinearity in the model. Hence, we deal with the nonlinear system under critical growth assumptions. In our work, we have overcome such difficulties applying specific discretization in time and proving regularity (continuity) of solutions at the discrete time level.

Objectives of the paper are: (i) rigorous thermodynamic derivation of the initial boundary value problem of coupled heat transport and water flow in variably saturated partially frozen porous media; (ii) following semi-discretization in time to prove the existence and regularity of variational solutions at the discrete time steps; (iii) comparison of numerical results with experimental data reported in literature.

Outline of the paper

In Section 2, we introduce our physical assumptions in the development of the model of heat and mass transfer processes in frozen and thawed porous media. In Section 3, based on rational thermodynamics, we present a complete derivation of the model, the balance equations completed by the set of constitutive and thermodynamic relationships and initial and boundary conditions. Rigorous formulation and mathematical analysis is presented in Section 4, in particular, mathematical preliminaries and formulation at the continuous level are treated in Section 4.1 and in Section 4.2. In Section 4.3, we specify our assumptions on data and modify structure conditions on coefficient functions. An application of the Rothe method of discretization in time leads to a coupled system of semilinear steady-state equations, which (together with the appropriate boundary conditions) form a semilinear degenerate elliptic boundary value problem, formulated in the form of operator equation in appropriate function spaces. The existence together with the regularity of the solution for this problem in space $W^{1,r}(\Omega)$ is proven in Section 4.4 using the general theory of coercive and pseudomonotone operators in Banach spaces and the regularity theory of elliptic problems with the mixed boundary conditions. Next, the problem is numerically resolved using the finite element method and, in Section 5, numerical results are performed to investigate the total water migration, ice and temperature distributions in the cylinder packed with sandy loam. Numerical results are compared with measured data reported by Mizoguchi [19], see also [6,7].

2. Physical assumptions

Variably-saturated porous materials consist of a solid skeleton, where the voids are partly filled by liquid water, air and, under freezing conditions, ice. According to the main physical processes in porous media under freezing-thawing conditions, the model is based on several hypotheses including that:

(i) partially frozen porous material is treated as a multi-phase system consisting of different α phases and components: solid skeleton ($\alpha = s$), liquid phase ($\alpha = \ell$), ice ($\alpha = i$) and gas;
(ii) capillary theory of the water flow is valid both in the unfrozen and frozen zones;
(iii) the solid skeleton and ice are rigid, i.e. the velocity of the solid particles and ice particles, respectively, is zero;
(iv) the gas (a mixture of dry air and water vapor) transfer has negligible effects on the heat and mass transport in unfrozen and partially frozen porous materials [20,21];
(v) all processes are single valued, i.e. hysteresis is not present in the characteristic curves;
(vi) local thermodynamic equilibrium exists between liquid water, solid skeleton and ice phases [22];
(vii) the generalized Clapeyron equation for thermodynamic equilibrium is used to relate changes in pressures and temperatures of ice and liquid water under frost action[1].

[1] Based on recent experimental observation [23], the combination of the water retention curve together with the Clapeyron equation during the freezing process underestimates the unfrozen water content in pores.