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Vibration analysis of electrostatically actuated nonlinear microbridges based on the modified couple stress theory

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ABSTRACT

In this paper natural frequency of electrostatically actuated microbridges is investigated based on the modified couple stress theory. Nonlinear formulation of Euler–Bernoulli microbeam is derived using Hamilton's principle. By considering the von-Karman strain, the nonlinearities caused by the mid-plane stretching are included in the formulation. To confirm the model, results of static deflection and natural frequency of microbeams are calculated using modified couple stress theory and compared to those evaluated based on the classical theory and experimental observations. At first, from experimental results of static deflection of a microcantilever, estimation for length scale parameter of polysilicon is presented. Using this value of length scale, natural frequency of microbridges under electrostatic actuation is calculated on the basis of the modified couple stress theory and compared with experimental results. The results of the modified couple stress theory are in very good agreement with experimental findings while the difference between the results predicted based on the classical theory and experimental findings is considerable.

1. Introduction

Microbemas are considered as an important part of many Micro-Electro-Mechanical Systems (MEMS) such as micropumps, microswitches, microresonators and micromirrors. A typical electro-statically actuated microresonator is consisting of a microbeam suspended above a fixed substrate actuated by electrostatic distributed load. Natural frequency of microresonators is very important and could be changed by external loads such as temperature, acceleration, force and pressure. According to the frequency shift, the value of these physical quantities (i.e. temperature, acceleration, force, pressure, etc.) can be measured.

Many researchers have studied the vibration behavior and natural frequency of resonant microbeams [1–7]. An experimental study on the natural frequency of polysilicon microdridges in vacuum is performed by Tilmans and Legtenberg [2]. Kalicinski et al. [3] presented a technique to determine the resonant frequency of microbeams based on the electric admittance measurements. Wang et al. [4] investigated the natural frequency and instability of multi-layer microbeams under electrostatic actuation. They examined the effect of residual stress and geometric parameters on the behavior of the microbeams. Experimental study on the resonant frequency of polysilicon and gold microbeams is performed by Zook

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and Bums [5] and also Atia and Hesto [6] respectively. Abdel-Rahman et al. [7] studied the static behavior and natural frequency of electrostatically actuated microbridges considering the geometric nonlinearities.

The structures used in MEMS have the dimensions in order of microns and sub-microns. Many experiments performed on the mechanical behavior of micro scale structures revealed that the classical theory underestimates the stiffness of these components [8–11]. These experiments also indicated that the normalized stiffness of small structures, which according to the classical theory should be independent of the structure size, is significantly size-dependent. Some of these experimental researches can be mentioned as below.

– Torsion of copper microbars by Fleck et al. [8].
– Bending of nickel microbeams by Stolken and Evans [9].
– Bending of epoxy microbeams by Chong and Lam [10].
– Deflection of microcantilevers made of polypropylene by McFarland and Colton [11].

In all of these experiments, the size-dependency of mechanical behavior of micro scale structures has been detected and it is indicated that the classical theory underestimates the stiffness of micro scale structures. Based on the above mentioned experiments, it is concluded that the classical theory is unable to predict the accurate stiffness of micro scale structures and model the size-dependency in these components. However, utilizing couple stress theory [12–15] the size-dependency of micro scale structures can be interpreted.

The couple stress theory has been introduced as a non-classical continuum theory by Koiter [12] and Mindlin and Tiersten [13] in 1960s. This theory considers the couple stresses as higher order stresses acting on an element of material. In this theory, beside the classical material constants (i.e. elastic modulus and Poisson's ratio) new material constants (i.e. length scale parameters) are introduced which relate the higher order stresses to the kinematic quantities. Based on this theory the size-dependency of mechanical behavior of micro scale structures has been successfully justified [14–16].

Yang et al. [17], developed the modified couple stress theory by employing a non-classical equilibrium equation (i.e. equilibrium equation of moment of couples). Many researchers employed this new theory to investigate the size-dependent behavior of micro scale structures. For example, Park and Gao [18] used this theory to interpret the experimentally detected size-dependent behavior of epoxy microbeams. Kong et al. [19] utilized the Hamilton's principle and developed an Euler–Bernoulli beam model based on the modified couple stress theory. Wang [20] utilized this theory to analyze dynamic behavior of fluid-conveying microtubes. Moreover, based on this theory, the size-dependent dynamic behavior of microcantilevers under suddenly applied voltage and the size-dependent yielding behavior of micro scale structures have been investigated by Rahaeifard et al. [21] and Kahrobaiyan et al. [22], respectively. Ma et al. [23] used this theory to derive the governing equation of Timoshenko beams. Furthermore, Xial et al. [24] and Asghari et al. [25], presented the size-dependent formulation of nonlinear Euler–Bernoulli and linear Timoshenko beams. The size dependent static behavior of microcantilevers and microbridges under electrostatic actuation, is investigated by Rahaeifard et al. [26,27]. They estimated the length scale of silicon based on the experimental results of static pull-in of microcantilevers [26].

For microbeams with immovable supports (such as microbridges), the nonlinearity caused by the mid-plane stretching has a considerable effect on the mechanical response of the beam. It has been shown that mid-plane stretching has a notable influence on the static and dynamic behavior of electrostatically actuated microbridges [7,27].

To sum up, since the classical continuum theory is unable to predict the mechanical behavior of micro scale structures, the non-classical continuum theories were proposed to fill this gap. Hence in this paper, due to the significance of size-dependency in micro scale structures, the size-dependent dynamic behavior of a nonlinear microbridge under electrostatic actuation is investigated. Based on the modified couple stress theory, the natural frequency of the nonlinear microbridge is calculated and compared with experimental results and those of classical theory. At first, the value of length scale of polysilicon is extracted based on the experimental results of static deflection of an electrostatically actuated microbeam. Using this value of length scale, natural frequency of microbridges under electrostatic actuation is calculated and compared with experimental results. The results of modified couple stress theory are in very good agreement with experimental observations while comparing the results given by the classical theory with experimental findings reveals that the classical theory gives a rough estimation of the mechanical behavior of these components.

2. Microbeam modeling

According to the modified couple stress theory, the strain energy density of an elastic continuum can be written as [19]

$$\bar{u} = \frac{1}{2}(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}) \quad (i,j = 1,2,3), \tag{1}$$

in which σ_{ij}, ε_{ij}, m_{ij} and χ_{ij} represent components of the stress tensor, strain tensor, the deviatoric part of the couple stress tensor and the symmetric curvature tensor, respectively. The deviatoric part of couple stress tensor is related to the symmetric curvature tensor as follows.

$$m_{ij} = 2l^2\mu\chi_{ij}, \tag{2}$$