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Numerical method to initial-boundary value problems for fractional partial differential equations with time-space variable coefficients

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ABSTRACT

In this paper, Haar wavelet operational matrix(HWOM) is proposed to solve initial-boundary value problems for a class of time-space fractional partial differential equations of Caputo sense with variable coefficients in both time and space

$$\sum_{i=1}^{n}\theta_i\frac{\partial^{\gamma_i}u(x,t)}{\partial t^{\gamma_i}}=v(x,t)\frac{\partial^{\alpha}u(x,t)}{\partial x^{\alpha}}+d(x,t)\frac{\partial^{\beta}u(x,t)}{\partial x^{\beta}}+q(x,t),$$
$$0<x<1,\quad 0<t\le 1, \tag{1}$$

as an extension of Rehman and Khan's (2013) work. We obtain a matrix L instead of Q_α in Rehman and Khan (2013). when dealing with boundary conditions. By utilizing the operational matrix of fractional integration and Hadamard product,we made an improvement of algorithm to deal with time-space coefficients and gave the error analysis of the HWOM for space-time dimensions. Some numerical results are paralleled with exact solutions to show the efficiency and precision of the presented technique.

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1. Introduction

The theory of fractional calculus is an old mathematical topic with history as long as that of integral ones. Fractional differential equations as an application of fractional calculus have recently proved to be valuable tools for the modeling of many phenomena. Comparing with integer order differential equations, the fractional ones provide an excellent instrument for description of memory and hereditary properties of various materials and processes [1]. Therefore, fractional calculus and fractional differential equations have been emerged in the last decades for many researchers in a wide area of applications over engineering [2–5], physics [6–9], mathematics [10–12],finance [13–15], biology [16] and other sciences. In order to obtain the analytical or numerical solutions for fractional differential equations, many methods are proposed, such as finite difference method [12,14,17–19], homotopy perturbation method [2,6,9,20], Adomian decomposition method [21–23], wavelet method [6,24–28] and so on.

Wavelets, a spacial kind of oscillatory functions with compact support, have received much attention in this field of partial differential equations. Since Chen and Hsiao [29] first proposed a Haar operational matrix for the integration of Haar function vectors and used it for solving differential equations, many works have been done. For example, Wang et al. [25] applied it into

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a special form of time-space fractional partial differential equations with constant coefficients. Wei et al. [26] and Chen and Wu [28] presented the HWOM of fractional differentiation to solve a class of space-time fractional convection-diffusion equations with variable coefficients. Erfanian and Gachpazan [30] solved a mixed nonlinear mixed Fredholm–Volterra integral equation by using the properties of rationalized HWOM which is defined from RH functions. Especially, Rehman and Khan [24] obtained solutions of boundary value problems for linear fractional partial differential equations by reducing the equations to Sylvester matrix equations with the help of Haar wavelets operational matrix. In addition, Stojanovic and Gorenflo [31] investigated the nonlinear two-term time fractional diffusion-wave problem and proved the existence and the uniqueness of the solution of the problem.

Motivated by above works, we will focus on solving a class of time-space fractional partial differential equations with variable coefficients in both time and space based on the Haar wavelet method. In the Eq. (1), $v(x, t)$, $d(x, t)$, $q(x, t)$ as coefficients are the known continuous functions, $u(x, t)$ is the unknown function, $0 < \theta_i, \alpha \leq 1, 1 < \beta \leq 2, \gamma_i > 0, i = 1, 2, \dots, n$ and $\sum_{i=1}^{n} \theta_i = 1$. $\frac{\partial^{\gamma_i} u}{\partial t^{\gamma_i}}, \frac{\partial^{\alpha} u}{\partial x^{\alpha}}, \frac{\partial^{\beta} u}{\partial x^{\beta}}$ are fractional derivative of Caputo sense [1,32]. Note that if $\alpha = \theta_1 = 1, v(x, t) = -b(x), d(x, t) = a(x)$, Eq. (1) is the time-space fractional convection-diffusion equation [28] solved by Haar wavelet method. Another kind of time fractional convection-diffusion equation with a source term was studied in [33,34] using discretization and RBFs approximation method if $v(x, t)$, $d(x, t)$ are constants and $\alpha = \theta_1 = 1, \beta = 2$. Zhou and Xu [35]derived and utilized the second kind Chebyshev wavelets operational matrix of integration to solve convection-diffusion equations as $\alpha = \theta_1 = 1, \beta = 2, v(x, t) = -a(x), d(x, t) = b(x)$. Also, some other researchers have worked on the fractional time-space advection-dispersion equations in [10,11,19] if only $\theta_1 = 1$ in Eq. (1).

The organization of this paper is as follows: In Section 2, we introduce some necessary definitions and mathematical preliminaries of fractional calculus theory. In Section 3, after describing the basic formulation of Haar wavelet, we derive the Haar wavelet operational matrix of fractional integration. In Section 4, we give the algorithm procession of this method. In Section 5, we present the error analysis of the HWOM for space-time dimensions. Several results and discussion are shown the efficiency and simplicity of the method in Section 6. Finally some conclusions are given in Section 7.

2. Preliminaries and notations

In this section, we give some necessary definitions and preliminaries of the fractional calculus theory which are required for establishing our results [1,32].

Definition 1. The Riemann–Liouville fractional integral operator of order $\alpha \geq 0$ is defined as

$$(I_x^{\alpha} u)(x, t) = \begin{cases} \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^x (x - \tau)^{\alpha - 1} u(\tau, t) d\tau, & \alpha > 0, \\ u(x, t), & \alpha = 0, \end{cases} \tag{2}$$

where $\Gamma(.)$ denoting the gamma function and its fractional derivative of order $\alpha \geq 0$ is given by

$${}_L D_x^{\alpha} u(x, t) = \begin{cases} \dfrac{1}{\Gamma(n - \alpha)} \dfrac{d^n}{dx^n} \displaystyle\int_0^x (x - \tau)^{n - \alpha - 1} u(\tau, t) d\tau, & 0 \leq n - 1 < \alpha < n, \\ \dfrac{d^n u(x, t)}{dx^n}, & \alpha = n \in N. \end{cases} \tag{3}$$

Definition 2. The fractional derivative of order $\alpha \geq 0$ in the Caputo sense is defined as

$$\frac{\partial^{\alpha}}{\partial x^{\alpha}} u(x, t) = \begin{cases} \dfrac{1}{\Gamma(n - \alpha)} \displaystyle\int_0^x (x - \tau)^{n - \alpha - 1} u^{(n)}(\tau, t) d\tau, & 0 \leq n - 1 < \alpha < n, \\ \dfrac{d^n u(x, t)}{dx^n}, & \alpha = n \in N. \end{cases} \tag{4}$$

The Caputo fractional derivative of order α is also defined as $\frac{\partial^{\alpha}}{\partial x^{\alpha}} u(x, t) = I_x^{n-\alpha} \frac{\partial^n}{\partial x^n} u(x, t)$. The useful relation between the Riemann–Liouville operator and the Caputo operator is given by the following expression:

$$I_x^{\alpha} \frac{\partial^{\alpha}}{\partial x^{\alpha}} u(x, t) = u(x, t) - \sum_{k=0}^{n-1} \frac{\partial^k u(0^+, t)}{\partial x^k} \frac{x^k}{k!}, t > 0, n - 1 < \alpha < n. \tag{5}$$

3. Haar wavelet and operational matrix of the fractional integration

3.1. Haar wavelet

For the Hilbert space $L^2[0, 1)$, the orthogonal basis $h_n(t)$ of Haar wavelet functions is defined as [29]:

$$h_0(t) = 1$$

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