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Fuzzy uncertainty propagation in composites using Gram–Schmidt polynomial chaos expansion

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A R T I C L E I N F O

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A B S T R A C T

The propagation of uncertainty in composite structures possesses significant computational challenges. Moreover, probabilistic descriptions of uncertain model parameters are not always available due to lack of data. This paper investigates on the uncertainty propagation in dynamic characteristics (such as natural frequencies, frequency response function and mode shapes) of laminated composite plates by using fuzzy approach. In the proposed methodology, non-intrusive Gram–Schmidt polynomial chaos expansion (GPCE) method is adopted in uncertainty propagation of structural uncertainty to dynamic analysis of composite structures, when the parameter uncertainties represented by fuzzy membership functions. A domain in the space of input data at zero-level of membership functions is mapped to a zone of output data with the parameters determined by D-optimal design. The obtained meta-model (GPCE) can also be used for higher α-levels of fuzzy membership function. The most significant input parameters such as ply orientation angle, elastic modulus, mass density and shear modulus are identified and then fuzzified. The proposed fuzzy approach is applied to the problem of fuzzy modal analysis for frequency response function of a simplified composite cantilever plates. The fuzzy mode shapes are also depicted for a typical laminate configuration. Fuzzy analysis of the first three natural frequencies is presented to illustrate the results and its performance. The proposed approach is found more efficient compared to the conventional global optimization approach in terms of computational time and cost.

1. Introduction

Composite materials have gained immense popularity in application of aerospace, marine, automobile and construction industries due to its weight sensitivity and cost-effectiveness. Such structures are prone to considerable uncertainty in their fiber and material parameters. During production of composite materials, it is always subjected to large variability due to unavoidable manufacturing imperfection, operational factors, lack of experience and precise test data. Therefore, it is important to investigate the structural behavior of composites due to the variability of parameters in each constituent laminate level. This information predicts the correlation between the dynamic characteristics, input-output variables and structural health. Typical uncertainties incurred are intra-laminate voids, incomplete curing of resin, excess resin between plies, excess matrix voids, porosity, variations in ply thickness and fiber parameters. In practice, an additional factor of safety is assumed by designers due to difficulty in quantifying those uncertainties. This existing practice of designer results in either an ultraconservative or an unsafe design.

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Uncertainty can be modeled either by probabilistic or non-probabilistic approach. Due to availability of limited sample experimental or testing data (crisp inputs), it will be more realistic to follow non-probabilistic approach rather than probabilistic approach. In recent years, significant advances have been made in representing uncertainty in composite material properties by probabilistic models. However, there has been little attention in the use of non-probabilistic models such as fuzzy. In most recent studies, the probability density function is used to portray the map of volatility in output characteristics while many procedures such as Monte Carlo simulation and probabilistic finite element method [1], random field [2] models, random matrix [3], and generalized polynomial chaos with Karhunen–Loève expansion [4] are employed. The propagation of structural uncertainty in laminated composite structures is carried out by probabilistic approach [5,6]. Following random variable based probabilistic approach stochastic analysis for free vibration responses of composite plates and shells are reported recently including the effects of uncertainties associated with twist angle, rotation and environmental factors such as temperature [7–9]. However, these methods usually need large volumes of data, which are expensive and computational costs are high. An alternative approach, assuming that large quantities of test data are not available, would be to use non-probabilistic methods such as interval and fuzzy. Fuzzy finite element analysis [10–11] aims to combine the power of finite element method and uncertainty modelling capability of fuzzy variables. One way to view a fuzzy input-output variable is the universality of an interval variable. It should be noted that the intervals do not represent the values of the variable, but the knowledge about the range of possible values that a variable can take. For an uncertain variable represented by interval, the values of the parameters can be observed within the two bounds (lower and upper). A membership function is introduced in fuzzy approach [12]. In real-life problems, original Monte Carlo simulation is expensive due to high computational time. Therefore, the aim of the majority of current research is to reduce the computational cost. Under the possibilistic interpretation of fuzzy sets [13] and uncertainty environment [14], fuzzy variables would become a generalized interval variables. Consequently techniques employed in interval analysis such as classical interval arithmetic [15], affine analysis [16] or vertex theorems [17] can be used. The Neumann expansion [18], the transformation method [19], and response surface based methods [20] are proposed for fuzzy analysis. In this context, recently fuzzy analysis is employed to deal with uncertainties in engineering problems using only available data [21]. Earlier fuzzy approach has been applied to safety analysis [22], random system properties [23] and optimal design [24]. In contrast, PCE approach [25] for material uncertainty effect on vibration control of smart composite plate and High Dimensional Model Representation (HDMR) approach are proposed [26] for the propagation of fuzzy uncertain variables through a complex finite element model. The fracture and fatigue damage response of composite materials is studied by using fuzzy [27] while the robust stabilization design of nonlinear stochastic partial differential systems by Fuzzy approach [28] and a fuzzy-probabilistic approach introduced for strain-hardening cement-based composites [29]. Of late, the modeling uncertainty for risk assessment is studied by an integrated approach with fuzzy set theory and Monte Carlo simulation [30]. The fuzzy logic based approach to FRP retrofit of columns [31] and grey-fuzzy algorithm [32] on composites are studied while experimental study with fuzzy logic modeling [33] is also investigated. The coupling of fuzzy concepts [34,35] and the modeling of arbitrary uncertainties using Gram–Schmidt polynomial chaos [36] can open a novel idea of research.

In general, Monte Carlo simulation technique is popularly utilized to generate the uncertain random output frequency dealing with large sample size. Although the uncertainty in material and geometric properties can be computed by direct Monte Carlo Simulation, it is inefficient and incurs high computational cost. Recently, the authors developed a fuzzy uncertainty propagation method [37] based on Legendre orthogonal polynomial chaos which was intrusively applied to static analysis of a rod with uncertain stiffness parameters. The present study employs the use of Gram–Schmidt algorithm based polynomial chaos expansion in propagation of structural uncertainties of composite structures, when parameter uncertainties are represented by fuzzy membership functions. In practice, the fuzzy models can be used when there is lack of data to estimate an accurate PDF of the uncertain parameters. This paper considers the application of the fuzzy propagation method to dynamic analysis of laminated composite plates with realistic uncertain parameters such as ply orientation. The polynomial chaos terms are determined by the method explained in Section 2. The obtained polynomial chaos expansion acts as a surrogate model (meta-model) for the full finite element model of composite structure. The regression coefficients of the PCE are then determined by first sampling in the space of input parameters (D-optimal design [38,39] in this paper) and then a least square technique. Since the widest range of input parameters are considered to be at zero membership function, the regression coefficients can be obtained once at this level and used for all highest α-cuts. The application of fuzzy PCE approach for uncertainty quantification in the field of composite structures is the first attempt of its kind to the best of authors' knowledge. The code which was developed by the authors in an earlier literature [40] is combined non-intrusively with the proposed method in this present study to treat uncertainty associated with complex systems like laminated composite structures. In the present study, four layered graphite–epoxy composite laminated cantilever plate is considered as furnished in Fig. 1.

2. Theoretical formulation

In the fuzzy concept [36], a set of transitional states between the members and non-members are defined via a membership function $[\mu_{p_i}]$ that indicates the degree to which each element in the domain belongs to the fuzzy set. The fuzzy number $[\tilde{p}_i(\tilde{\omega}_\alpha)]$ considering triangular membership function can be expressed as,

$$\tilde{p}_i(\tilde{\omega}_\alpha) = [p_i^U, p_i^M, p_i^L], \tag{1}$$

where p_i^M, p_i^U and p_i^L denote the mean value, the upper bound and lower bounds, respectively. $\tilde{\omega}_\alpha$ indicates the fuzziness corresponding to α-cut where α is known as membership grade or degree of fuzziness ranging from 0 to 1.