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Time-domain simulation of second-order irregular wave diffraction based on a hybrid water wave radiation condition

G. Xu [a,c], A.M.S. Hamouda [a,*], B.C. Khoo [b]

[a] Department of Mechanical and Industrial Engineering, Qatar University, Qatar
[b] Department of Mechanical Engineering, National University of Singapore, Singapore
[c] School of Naval Architecture and Ocean Engineering, Jiangsu University of Science and Technology, China

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A B S T R A C T

A time-domain second-order method is presented to simulate three-dimensional wave–body interaction. In the approach, Taylor series expansions are applied to the free surface boundary conditions, and a Stokes perturbation procedure is then used to establish the corresponding boundary value problem at first order and second order on the time-independent surfaces. A constant boundary element method, based on a Rankine source, is used to calculate the wave field at each time step. A proposed hybrid radiation condition, which is a combination of the multitransmitting formula and the damping zone, is studied to minimize the wave reflection, a stable integral form of the free-surface boundary condition is used to update the velocity potential on the free surface, and an auxiliary function is used to calculate high-order derivatives. The proposed model is first validated by linear irregular wave diffraction and is then applied to compute the second-order irregular Stokes wave diffraction with three wave components. It is shown that long time simulation can be performed with stability and accuracy and that the model can be used to simulate nonlinear irregular wave–structure interaction.

1. Introduction

It is well known that second-order effects may be important for the nonlinear hydrodynamic problem arising in many aspects of ocean engineering. Numerous studies have been published to date [1–7]. However, for the time-domain numerical simulation, the solution generally requires truncation of the fluid domain at some finite distance. There is no exact nonreflecting boundary condition for the truncated domain surface. For a long time simulation, an appropriate and effective radiation condition should be imposed on the so-called artificial boundary to minimize any wave reflection. This is a common problem faced in the numerical modeling of wave propagation. Various techniques have been developed to satisfy the radiation condition. Newman [8] introduced a theoretical method to absorb reflected waves on the sides of the closed basin for a linear potential; however, it is hard to apply the method to the analysis of the nonlinear case. Orlanski's method [9] and the absorbing beach scheme [10] have generally been implemented to model the open boundary. However, Orlanski's method may produce incorrect phase velocity if the mesh size is not sufficiently small near the open boundary. Clément [11] proposed a coupling method (piston-beach hybrid absorber) to absorb the wave, whereas Boo [12] used a numerical scheme which combines an absorbing beach and the stretching technique [13] to simulate the open boundary. Wang and Wu [7] imposed a radiation condition through a combination of the damping zone and the Sommerfeld–Orlanski equation. Clamond et al. [14] introduced a numerical beach to damp the (scaled)

* Corresponding author. Tel.: +974 55360976; fax: +974 44034122.
E-mail address: hamouda@qu.edu.qa (A.M.S. Hamouda).

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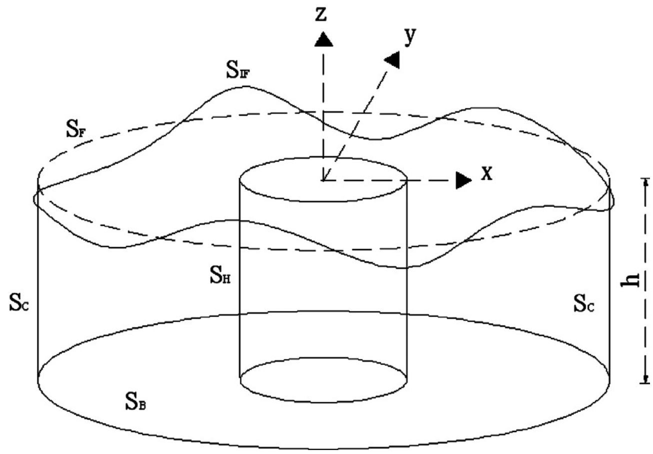

Fig. 1. Definition sketch.

tangential velocity at the free surface, with all the frequencies being damped with the same intensity. However, the efficiency of the damping zone method strongly depends on the ratio between the width of the beach and the length of the wave.

To find a more efficient and effective radiation condition to deal with the nonlinear irregular wave problem in the time domain, a multitransmitting formula (MTF) [15] used as the radiation condition for a water wave field is introduced. It has already been used in a finite element method to achieve the nonreflecting condition in the field of seismology by Liao [16], and has proven to be effective in the simulation of the earthquake waves. On the basis of the MTF method, the values of the diffracted velocity potential at certain positions (named the "transmitting layers") in the inner fluid domain [17] are obtained at each time step. Then the velocity potentials on the artificial boundary at the present time step can be formulated by potentials at prior time steps on the transmitting layers. In the MTF method, an artificial wave velocity is used to replace the actual wave velocity. Usually, it is not necessary to make the artificial wave velocity equal to the actual wave velocity. As a result, the flexibility of the method is very useful for dealing with the irregular wave problem. However, the MTF method can transmit waves only out of the artificial boundary effectively when the artificial wave velocity is in a certain range of the given actual wave velocity. Numerical tests in this work will show that a weak wave reflection from the open boundary is usually nonnegligible for irregular wave diffraction by the MTF method. As such, an improvement in treating this radiation condition becomes essential. On examining a variety of approaches, one finds that the hybrid method with a damping zone is an effective approach to eliminate the problem. Fairly similar findings have also been shown by Xu [18], Duan and Zhang [19], and Zhang and Duan [17].

Theoretically, numerical calculation based on the boundary element method can benefit much from the MTF approach compared with the damping zone approach [17]. Firstly, only one coefficient, named the "artificial velocity," needs to be considered in the MTF. The artificial velocity is a rough estimation of the actual wave velocity, which is usually fairly easy to obtain. Secondly, fewer elements are needed to be distributed on the fluid boundary in comparison with the damping zone. Thirdly, the numerical implementation is simpler than that of other radiation conditions. Finally, extension to nonlinear problems is possible—for example, second-order nonlinear [18,20,21] and fully nonlinear [19] problems. The MTF method has also been successfully used for simulation of harmonic wave radiation and diffraction [22,23] and irregular wave diffraction [24]. The first attempt to simulate second-order irregular wave diffraction was published by Xu and Hamouda [25]. In their article, only the hybrid water wave radiation condition to simulate the nonlinear wave–structure interaction was mentioned; however, the convergent study of artificial water wave velocity applied in the MTF method and the hybrid condition was not presented, especially for the irregular wave diffraction cases. In the present work, a detailed expression of two auxiliary functions and the integral form of the free-surface boundary condition is introduced where the values of the velocity potential on the free surface are easily estimated at each time step, and which shows excellent stabilities in our various numerical simulations for both linear and second-order cases. On the basis of the linear water wave diffraction theory, the stability of the MTF method has been studied. Our hybrid model is first verified by the linear irregular wave diffraction and the results obtained are compared with the frequency-domain solution. We find the effective length of the damping zone which suffices for the hybrid method. Next, the model is applied to the time-domain computation of the second-order Stokes wave diffraction of a bottom-mounted circular cylinder at finite water depth and the second-order irregular Stokes wave diffraction of a truncated cylinder at infinite water depth for both the high-frequency and the low-frequency wave–structure interaction cases. The model has been found to be accurate and efficient.

2. Mathematical formulation

The reference system of Cartesian coordinates is defined with the (x, y) plane coinciding with the still water surface S_F and z pointing vertically upward from the still water level as shown in Fig. 1. The body located at the center of the domain is rigid and fixed, and the instantaneous wetted body surface is denoted by S_H and its unit normal vector directed outward from the fluid region is denoted by $\boldsymbol{n}$. The seabed S_B is assumed to be horizontal along the plane at $z=-h$. Let t denote time and η be the