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An adaptive method to parameter identification and synchronization of fractional-order chaotic systems with parameter uncertainty

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ABSTRACT

In this paper, parameters of a fractional-order chaotic system are identified via a robust recursive error prediction method in presence of uncertainty. A generalized ARX structure has obtained by discretization of a continuous fractional-order differential equation defines the identification model. After identifying parameters of system, we use concept of active control method to synchronize two identified fractional-order chaotic systems. The validity of results are demonstrated through an example and also compared with other method.

1. Introduction

Fractional-order calculus is an area of mathematics that deals with derivatives and integrals from non-integer orders. In other words, it is a generalization of the traditional calculus that leads to similar concepts and tools, but with a much wider applicability. In the last two decades, fractional calculus has been applied in an increasing number of fields. One of the important applications of fractional calculation is chaotic systems. Chaotic systems are deterministic and nonlinear dynamical systems which are exponentially sensitive to initial conditions. This type of sensitivity is popularly known as the butterfly effect [1]. The chaotic dynamic of fractional-order systems is an important topic of study in nonlinear dynamics. In the last few years, this area of research has been growing rapidly [2,3]. Existence of chaos in fractional-order systems is conceived by Grigorenko and Grigorenko in 2003 [4]. They have investigated chaotic behavior in fractional-order Lorenz system and then many papers are published dealing with the chaotic behavior in fractional-order systems [5,6]. Parameters play an important role in chaotic systems and chaos synchronization. Parameter identification of integer-order chaotic systems has attracted much interest [7–9]. Many synchronization methods are valid for fractional-order chaotic systems with known parameters [10–12]. However, it is difficult to achieve synchronization and identify the parameters in the fractional-order chaotic systems with unknown parameters. To the best of our knowledge, there have been no results on parameter identification of fractional-order chaotic systems based on chaos synchronization because the design of controller and the updating law of parameter identification is a task with technique and sensitively depends on the considered systems. Many evolutionary solutions are proposed to identify the parameters and order of fractional-order chaotic systems [13–16]. All of these methods have weak performance when parameters have

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some uncertainties. Some other methods for parameter identification have proved the convergence of estimated parameter to its nominal value [17]. So we investigate recursive error predictor algorithm to identify parameters of system because this method is not only highly robust against the uncertainty and perturbation but also identifies unknown parameters quickly related to other methods. In the proposed method we explain adaptive recursive least square algorithm (ARLS) for parameter identification of fractional-order chaotic system when a parameter has uncertainty.

2. Preliminaries

2.1. Fractional calculus

2.1.1. Definition

To discuss fractional-order chaotic systems, we usually need to solve fractional-order differential equations. For the fractional differential operator, there are three commonly used definitions: Grünwald–Letnikov (GL) definition, Riemann–Liouville (RL) definition and Caputo definition. The GL definition of non-integer operator is given as:

$$D_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{(t-a)/h} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{1}$$

where $\binom{\alpha}{j} = \frac{\alpha(\alpha-1)\dots(\alpha-j+1)}{j!}$.

Captuo definition is given as below:

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{n-\alpha+1}} d\tau. \tag{2}$$

Also RL definition of fractional-order operator is described by:

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{n-\alpha+1}} d\tau, \tag{3}$$

where n is an integer such that $n-1 < \alpha < n$ and $\Gamma(.)$ is the Gamma function.

2.1.2. Some properties of fractional calculus

Throughout this paper we use the notation D^{α} as the simplified form of ${}_0^C D_t^{\alpha}(t > 0)$. In addition, we denote the fractional integral of order $\alpha > 0$ by $D^{-\alpha}$. In the following, we list some basic properties of fractional derivatives and integrals which are helpful in the proving analysis of the proposed method [12,13].

(1) For $\alpha = n$, where n is an integer, the operation $D^{\alpha} f(t)$ gives the same result as classical calculus of integer order n. In particular, when $\alpha = 1$, the operation $D^1 f(t)$ coincides with the ordinary derivative $\frac{d}{dt}(f(t))$.

(2) For $\alpha = 0$, the operation $D^0 f(t)$ is the identity operation:

$$D^0 f(t) = f(t). \tag{4}$$

(3) Similar to integer-order calculus, fractional differentiation and fractional integration operators are linear operators:

$$D^{\alpha}(af(t) + bg(t)) = aD^{\alpha} f(t) + bD^{\alpha} g(t). \tag{5}$$

(4) For $\alpha \geq 0$, the following equation holds:

$$D^{\alpha} D^{-\alpha} f(t) = D^0 f(t) = f(t), \tag{6}$$

which means that for the same fractional-order α, the fractional differentiation operator is a left inverse of the fractional integration operator.

(5) Suppose f has a continuous kth derivative on $[0, t](k \in N, t > 0)$, and let, $\alpha, \beta > 0$ be such that there exists some $l \in N$ with $l \leq k$ and $\alpha, \alpha + \beta \in [l-1, l]$, then,

$$D^{\alpha} D^{\beta} f(t) = D^{\alpha+\beta} f(t). \tag{7}$$

Note that the condition requiring the existence of the number l with the above restrictions in the property is essential. In this paper, we consider the case that, $\alpha, \beta \in (0, 1]$ and $\alpha + \beta \in (0, 1]$. Under these conditions property (5) holds.

2.1.3. Stability theorem of fractional-order system

In this section, some important results of the stability theorems for fractional-order systems are reviewed [10]. Consider the following linear system of fractional differential equation:

$$D^{\alpha} x = Ax, x(0) = x_0, \tag{8}$$

where $x \in R^n$, $A \in R^{n \times n}$, and $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_i, \dots, \alpha_n]^T$, $(0 < \alpha_i \leq 1)$ for $(i = 1, 2, \dots, n)$ is the fractional orders.