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Solution of multiple confocally elliptical layers with dissimilar properties in antiplane elasticity with eigenstrains and remote loading

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ABSTRACT

This paper provides a general solution for the problem of multiple confocally elliptical layers in antiplane elasticity. In the problem, the elastic medium is composed of many confocally elliptical layers with different elastic properties, and it is assumed to be isotropic. The eigenstrain is placed in the inclusion. The complex potentials for the inclusion and many layers are assumed in a particular form with two undetermined coefficients. The continuity conditions for the traction and the displacement along the interfaces are formulated exactly. From those conditions, the relation between two sets for two undetermined coefficients in two adjacent layers can be evaluated. By using the above-mentioned relation, a definite relation among (1) the remote loading, (2) the stress in inclusion and (3) the eigenstrian in inclusion can be evaluated. Within the three components, two of them are independent. The proposed condition of the uniform remote stress and the eigenstrain admits an internal uniform stress field. Many computed results are provided, which give a relation among the remote loading, the eigenstrain and stress in inclusion.

1. Introduction

Since the presence of inclusions in matrix affects the elastic field significantly, the study for the inclusion problem attracts many researchers. From a viewpoint of engineering application, the study of inclusion problem plays an important role in the advanced materials used for aerospace and other applications. From a viewpoint of theoretical analysis, the study of inclusion problem is more difficult than that for the isotropic medium since some conditions along interface are generally involved in the problem.

In earlier years, Eshelby studied an inclusion problem with prescribed eigenstrains [1]. If the uniform eigenstrains were applied in an inclusion, Eshelby proved that the elastic field in an ellipsoidal inclusion was also uniform. Similar situation holds in the Eshelby inclusion problem in plane elasticity. Later, the eigenstrain problems were studied in more detail [2]. Mura proposed the following catalog for the inclusion problems, (1) inhomogeneities, (2) homogeneous inclusions and (3) inhomogeneous inclusions. Since the eigenstrains and dissimilar elastic properties are involved in the inhomogeneous inclusions, the study for the problem has become a more difficult one.

A general solution was provided to the problem of an elastic elliptical inclusion under arbitrary antiplane loading [3]. The analysis in the solution was based upon antiplane complex potentials and facilitated by the use of conformal mapping technique

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and Laurent series expansion method. The antiplane shear problem of an elliptical inclusion embedded in an infinite medium subjected at infinity to a uniform stress field was studied [4]. An explicit general solution of an infinitely extended plate containing any number of circular inclusions under antiplane deformation was provided [5]. The suggested technique can be used for the circular inclusions only. It was proved that a three-phase elliptical inclusion under uniform remote stress and eigenstrain in antiplane shear admits an internal uniform stress field provided that the interfaces are two confocally ellipses [6]. A simple and effective method was used to derive the elastic field of the infinite homogeneous isotropic medium with two circular cylindrical inhomogeneities under the antiplane shear [7].

An interaction problem of arbitrarily distributed elliptical inclusions under longitudinal shear loading was considered [8]. An analytical solution in infinite series form for two circular cylindrical elastic inclusions embedded in an infinite matrix in antiplane shear was derived [9]. A novel efficient procedure to analyze the two-phase confocally elliptical inclusion embedded in an unbounded matrix under antiplane loadings was provided [10]. The analytical continuation method together with an alternating technique was used to derive the general forms of the elastic fields in terms of the corresponding problem. The null-field integral equation for an infinite medium containing circular holes and/or inclusions was derived [11]. Using the suggested null-field integral equation, multi-inclusion problem under antiplane shear was solved numerically.

Ting and Schiavone considered an anisotropic elastic inclusion of arbitrary shape embedded inside an infinite dissimilar anisotropic elastic medium (matrix) subjected to a uniform antiplane shear loading at infinity [12]. More recently, in the case of the remote tractions and linear distribution of eigenstrains, a closed form solution for Eshelby's elliptical inclusion in antiplane elasticity was obtained [13].

For the inclusion problem in plane elasticity, we can find a lot of papers in this field. A generalized and unified approach was presented to the two-dimensional plane problem of an elliptical inhomogeneity in an isotropic elastic medium [14]. The analysis is based upon the use of conformal mapping and Laurent series expansion of Muskhelishvili's complex potentials. The complex potentials in the matrix and the inclusion were expanded in the Laurent series. Wang considered the internal stress field of a three-phase elliptical inclusion bonded to an infinite matrix through an interphase layer when the matrix was subjected to remote uniform stresses [15].

The problem for a dissimilar elastic inclusion embedded in the infinite matrix was solved by using the complex variable boundary integral equation [16]. The boundary value problem for a finite plate containing two dissimilar inclusions was studied [17]. The matrix and the two inclusions have different elastic properties. The problem is solved by using the complex variable boundary integral equation (CVBIE). A comprehensive survey of recent works on inclusion problems was provided [18]. The problems of a single inclusion, two inclusions and multiple inclusions were discussed.

The interaction of a screw dislocation with a multicoated circular inclusion was studied [19]. The long-range elastic interaction of a line dislocation with multiple multicoated two-dimensional inclusions of arbitrary shape was studied [20]. Approximate closed-form solutions are found for two particular situations.

Null-field approach for the antiplane problem with elliptical holes and/or inclusions was suggested [21]. This method belongs to one kind of meshless methods since only collocation points on the real boundary are required. Similarly, null-field integral approach for the piezoelectricity problems with multiple elliptical inhomogeneities was suggested [22].

Recently, a general solution for a crack embedded in an inclusion of multiply confocally elliptical layers in antiplane elasticity was provided [23]. In the problem, the elastic medium is composed of a cracked inclusion, many confocally elliptical layers and the infinite matrix with different elastic properties. In addition, the remote loading is applied at infinity.

It is found that most previous studies in the solution of confocally elliptical layers problem were incomplete. For example, the previous publications only devoted to the inhomogeneity for one confocally elliptical layer with inclusion and matrix.

This paper provides a general solution for the problem of multiple confocally elliptical layers in antiplane elasticity. In the problem, the elastic medium is composed of many confocally elliptical layers with different elastic properties and it is assumed to be isotropic. In addition, the eigenstrain is placed in the inclusion. The complex potentials for the inclusion and many layers are assumed in a particular form with two undetermined coefficients. The conditions along the interfaces between two layers are proposed. From the imposed condition, the exact relation between two sets of two undetermined coefficients for jth layer and j+1th layer can be evaluated. Similarly, the conditions along the inclusion and the first layer can be formulated. Using those conditions, the relation between (1) two undetermined coefficients for the first layer and (2) the stress and the eigenstrain in the inclusion can be evaluated. By using the above-mentioned relations, a definite relation among (1) remote loading, (2) stress in inclusion and (3) eigenstrian in inclusion can be evaluated. Within the three components, two of them are independent. The proposed condition of the uniform remote stress and the eigenstrain admits an internal uniform stress field. Many computed results are provided, which give a relation among the remote loading, the eigenstrain and stress in inclusion.

This paper is an extension of the previous publication [13]. In [13], we only considered the inclusion with eigenstrain. In addition, the inclusion and the matrix have the same shear modulus of elasticity G. However, in this paper we consider the problem for one inclusion with confocally elliptical N-layers. Particularly, the shear modulus of the inclusion and many layers are different and they are denoted by G_o, G_1, G_2 and G_N.

The studied topic and suggested formulation in this paper have a lot of difference with respect to those in [23]. First, we studied the problem of crack embedded in the inclusion in [23]. Therefore, the interested and computed results are the stress intensity factor at the crack tip. However, this paper aims to study the stress field for the multiple confocally elliptical layers with dissimilar properties in antiplane elasticity with eigenstrains and remote loading. Particularly, the interaction between the eigenstarin ε^*_{xz} (or ε^*_{yz}) and remote loading σ^∞_{xz} (or σ^∞_{yz}) are studied.