Applied Mathematical Modelling 40 (2016) 4612–4624

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A simple parametric method to generate all optimal solutions of fuzzy solid transportation problem

Hale Gonce Kocken*, Mustafa Sivri

Department of Mathematical Engineering, Faculty of Chemical and Metallurgical Engineering, Yildiz Technical University, TR-34220 Davutpasa, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 4 May 2012
Revised 7 April 2015
Accepted 30 October 2015
Available online 9 December 2015

Keywords:
Solid transportation problem
Fuzzy mathematical programming
Parametric Programming

ABSTRACT

This paper deals with the fuzzy solid transportation problem (FSTP) that has fuzzy cost coefficients, fuzzy supplies, fuzzy demands and fuzzy conveyances. All these fuzzy quantities of FSTP are assumed to be triangular fuzzy numbers. For this problem, we propose an approach to generate all optimal solutions parametrically. The first stage of our approach is to determine the feasibility range based on fuzzy supply–demand–conveyance quantities. In the second stage, the breaking points of fuzzy costs are found by intersecting the membership functions of the fuzzy costs. The last stage constructs the optimal solutions of FSTP by means of some proposed auxiliary programs. Also a numerical example has been provided to illustrate our solution procedure.

1. Introduction

Transportation models play an important role in logistics and supply chain management for reducing cost and improving service. The classical transportation problem (TP) is a special type of linear programming problem. The purpose of TP is to transport the goods from sources to destinations. TP is also used in inventory control, manpower planning, personnel allocation, etc.

The solid transportation problem (STP) is a generalization of the classical TP. The necessity of considering this special type of TP arises when there exist different types of products and also when heterogeneous transportation modes called conveyances are available for the shipments of goods. Thus, three item properties (called parameters) are taken into account in STP instead of two (source and destination). The source quantities (a_i) may be production facilities, warehouses or supply points whereas the destination quantities (b_j) may be consumption facilities, warehouses, sales outlets or demand points. And the conveyances may be trucks, air freights, freight trains or ships. In practice, the parameters of TP or STP are not always exactly known and stable. This imprecision may follow from the lack of exact information, changeable economic conditions, uncontrollable factors, the nature of the parameters, etc. A frequently used way of expressing the imprecision is to use the fuzzy numbers. It enables us to consider tolerances for the model parameters in a more natural and direct way. Therefore, TP or STP with fuzzy parameters seems to be more realistic and reliable.

For TP, Chanas and Kuchta [1] proposed a concept of the optimal solution of the transportation problem with fuzzy cost coefficients and an algorithm determining this solution. Das et al. [2] focused on the solution procedure of the multi-objective version of TP where all the parameters have been expressed as interval values by the decision maker (DM). Ahlatçıoğlu et al. [3] proposed a model for solving the transportation problem that supply and demand quantities are given as triangular fuzzy

* Corresponding author. Tel.: +902123834605; fax: +902123834590.
E-mail addresses: hgonce@yildiz.edu.tr, halegk@gmail.com (H.G. Kocken), msivri@yildiz.edu.tr (M. Sivri).

http://dx.doi.org/10.1016/j.apm.2015.10.053

numbers bounded from below and above, respectively. Basing on extension principle, Liu and Kao [4] developed a procedure to derive the fuzzy objective value of the fuzzy transportation problem where the cost coefficients, supply and demand quantities are fuzzy numbers. Using signed distance ranking, defuzzification by signed distance, interval-valued fuzzy sets and statistical data, Chiang [5] gets the transportation problem in the fuzzy sense. Ammar and Youness [6] examined the solution of multi-objective TP which has fuzzy cost, source and destination parameters. They introduced the concepts of fuzzy efficient and α-parametric efficient solutions. And Barough [7] presented a two stage procedure for fuzzy transportation problem in which the cost coefficients and supply and demand quantities are fuzzy numbers. Ojha et al. [8] formulated single and multi-objective transportation models with fuzzy relations under the fuzzy logic. In that paper, the parameters of models are stated by verbal words such as 'very high', 'high', 'medium', 'low' and 'very low'. And both models are solved with real coded Genetic Algorithms.

For STP, Jimenez and Verdegay [9] propose a Genetic Algorithm based solution method to FSTP in the case in which the fuzziness affects only in the constraint set and a fuzzy solution required. And they improve this paper in Jimenez and Verdegay [10]. Also in 1998, they analyzed two uncertain models for the STP in the names of interval and fuzzy STP [19]. Both models are extensions of interval and fuzzy TP respectively in which three item properties are assumed. Liu [11] develops a method that is able to derive the fuzzy objective value of the FSTP. Based on the extension principle, the FSTP is transformed into a pair of mathematical programs that are employed to calculate the lower and upper bounds of the fuzzy total transportation cost at possibility level α. From different values of α, the membership function of the objective value is approximated. In Ojha et al. [12], a multi-objective solid transportation problem is considered with generalized fuzzy transportation costs. In the proposed problem the objective functions are expressed in fuzzy equality sense through a possibility measure and the entropy function was considered as an additional objective function. Ojha et al. [13] considered a STP for an item with fixed charge, vehicle cost and price discounted varying charge. To solve the problem, genetic algorithm which is based on roulette wheel selection, arithmetic crossover and uniform mutation were suitably developed and applied. Ojha et al. [14] investigate the best optimal policy for a multi-criteria solid transportation problem with nested discounts in transportation costs. Using multi-objective genetic algorithm, first a set of pareto optimal solutions is obtained and then the best one solution is chosen using AHP.

In this paper, we focus on the solution procedure of the solid transportation problem with fuzzy parameters, i.e. fuzzy cost coefficients, fuzzy supply, fuzzy demand and fuzzy conveyance quantities. Because of its fuzzy parameters, this transportation problem is very complicated and also due to the fuzziness in the costs it has a non-linear structure. To overcome these difficulties, we give an approach that generates all possible solutions of FSTP with regard to cuts of fuzzy parameters. Also, we note that our method works only for single objective function case and the fuzzy parameters in given triangular forms. In the first stage of our method, the feasibility range based on fuzzy supply–demand quantities is obtained. Then, breaking points of cost coefficients (i.e. the possible values of cost-satisfaction level that can change the optimal solution set) are found by intersecting the membership functions of cost coefficient. Finally, considering all the breaking points and the feasibility range, the optimal solutions of FSTP are constructed by some proposed auxiliary programs which are based on parametric programming techniques. A numerical example has been provided to illustrate our solution procedure.

This paper is organized as follows. After having presented brief information about fuzzy mathematics in the next section, the mathematical model of FSTP is given in Section 3. Section 4 introduces the proposed procedure with three stages. Section 5 gives an illustrative numerical example. Finally, Section 6 includes some results.

2. Preliminaries

In this paper, we assumed that the parameters of FSTP are expressed as triangular fuzzy numbers. In this section, brief information about the fuzzy numbers especially triangular fuzzy numbers are presented. For more detailed information, the reader should check Zimmermann [15].

Definition 2.1. A fuzzy number $\tilde{a}$ is an upper semi-continuous normal and convex fuzzy subset of the real line R.

Definition 2.2. A fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number (TFN) if its membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x < a_2 \\ 1, & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x \le a_3 \\ 0, & x > a_3 \end{cases}, \tag{1}$$

where $a_1, a_2, a_3 \in R$ and $a_1 \le a_2 \le a_3$. The figure of the fuzzy number $\tilde{a}$ is given in Fig. 1. In this paper, we called these ordered elements as characteristic points of $\tilde{a}$.

Some algebraic operations on TFNs that will use in this paper are defined as follows:

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs.